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The Principal Principle and Posterior Credences†

Ilho Park‡

In this paper, I will show that, contrary to what many philosophers of chance have thought, Lewis's original Principal Principle itself does not stay silent on how our posterior credences should be related to chances. Furthermore, I will prove, with the help of this result, that Nissan-Rozen (2013) fails to show that the Principal Principle is not preserved under Jeffrey Conditionalization. Indeed, the Principal Principle is preserved under any coherent belief updating rule.

【Keywords】 Principal Principle, initial credences, posterior credences, Jeffrey Conditionalization

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‡ Chonbuk National University, ipark.phil@gmail.com.

1. Introduction

Nissan-Rozen (2013) recently provided an interesting argument about the relationship between the Principal Principle and Jeffrey Conditionalization. According to his argument, the Principal Principle is not preserved under Jeffrey Conditionalization. That is, even if an agent's rational initial credence function satisfies the Principal Principle, her posterior credence function obtained by Jeffrey Conditionalization on a partition cannot satisfy the Principal Principle. In this paper, however, I will show that Nissan-Rozen's argument fails to prove the non-preservation at issue. In particular, it will be shown that his failure is due to a wrong formulation of the relation between the Principal Principle and posterior credences.

As is well known, Lewis's original Principal Principle concerns how chances should be related to an agent's rational initial credence function. By 'an agent's rational initial credence function', he means the coherent credence function the agent has before any course of experience. As is also well known, the principle can be formulated in at least two ways (see Lewis 1980, p. 266, p. 277). Here are the two formulations:[1]

PP_0^-: For any proposition A, $C_0(A|XE) = x$,
where C_0 is an agent's coherent initial credence function, X is the proposition that the chance, at a time, of A is x and E is admissible with respect to X.

PP_0 : For any proposition A, $C_0(A|TH) = ch_{TH}(A)$,
where C_0 is an agent's coherent initial credence function,

[1] Notational Remarks: Here and below, '¬A' refer to the negation of A and 'AB' to the conjunction of A and B.

T is the complete theory of chance at a world, H is the complete history up to a time at the world and ch_{TH} is a chance distribution that is obtained at the time and the world.

A crucial difference between the two formulations is that, while PP_0^- depends on the notion of admissibility, PP_0 does not. Roughly speaking, "admissible propositions are the sort of information whose impact on credence about outcomes comes entirely by way of credence about the chances of those outcomes."[2] That is, an agent's knowing admissible propositions could be entirely represented by the change of her credences in the associate chances. As well known, Lewis does not provide any strict definition of admissibility. Rather, he just suggests some sufficient conditions for it—for example, the propositions containing historical information and/or hypothetical information are admissible relative to the relevant chance propositions. Despite the difference between PP_0^- and PP_0, the above two formulations are equivalent to each other (see Meacham (2010) for the relevant discussion).[3]

As I mentioned, Nissan-Rozen (2013) argues that the Principal Principle is not preserved under Jeffrey Conditionalization. In particular, he argues that, when an agent's rational initial credence function that satisfies PP_0^- is updated by Jeffrey Conditionalization on a partition, her posterior credence function obtained so cannot

2) Lewis (1980), p. 272.

3) Strictly speaking, PP_0 is weaker than PP_0^-. Note that X is equivalent to a disjunction of all T_iH_is such that $ch_{T_iH_i}(A)=x$. According to Lewis (1980, p. 279), PP_0 does not imply PP_0^- when the number of disjuncts in question is infinite. However, he also says that this is unlikely to matter. I agree.

satisfy the Principal Principle. Of course, a similar argument can be made using PP_0, rather than PP_0^-. In the next sections, I will provide such an argument and critically examine it.

2. Nissan-Rozen's Argument

Here we should note again that the Principal Principle *itself* concerns only an agent's rational initial credence function. Thus, in order to argue that the Principal Principle is not preserved under Jeffrey Conditionalization, it should be explained how the Principal Principle constrains the posterior credence function updated by Jeffrey Conditionalization. Regarding this, Nissan-Rozen (2013, p. 841) specifically addresses two reasons for which Lewis refers to an agent's 'rational initial credence function'. The first is that the initial function is regular, in the sense that the function assigns zero only to the empty proposition.[4] The second is that the initial function is one that the agent has before learning any inadmissible proposition. In light of this consideration, he concludes that an agent's credence function at time t should obey the following principle:

PP_t^-: For any proposition A, $C_t(A|XE) = x$,
where C_t is an agent's regular coherent credence function, X is the proposition that the chance, at a time, of A is x and E is admissible with respect to X.

Note that, when C_0 is regular as well as coherent and C_t is obtained

4) Here, 'empty propositions' refer to the propositions that is false at any possible world.

from C_0 by Jeffrey Conditionalization on a partition, C_t obtained so is also regular as well as coherent. Moreover, if C_t is regular, then the agent who has C_t does not learn any inadmissible proposition (I assume that if an agent learns a proposition, then her credence in the proposition is 1). According to Nissan-Rozen, thus, if the Principal Principle is preserved under Jeffrey Conditionalization, it should hold that:

> **PRESERVATION$^-$**: When an agent's rational initial credence function C_0 obeys PP_0^- and C_t is updated from C_0 by Jeffrey Conditionalization, C_t obeys PP_t^-.

Nissan-Rozen proves, however, that PRESERVATION$^-$ cannot be the case.

As I have said, PP_0^- and PP_0 are equivalent to each other. So, a parallel argument can be presented using PP_0. Note that, unlike PP_0^-, PP_0 does not make use of the notion of admissibility. Thus, the parallel argument may be more convenient for examining Nissan-Rozen's argument.

In order to provide the argument in question, we should first formulate a principle that corresponds to PP_t^-. Here is such a principle:

> PP_t: For any proposition A, $C_t(A|TH) = ch_{TH}(A)$, where C_t is an agent's regular coherent credence function, T is the complete theory of chance at a world, H is the complete history up to a time at the world and ch_{TH} is a chance distribution that is obtained at the time and the world.

Note again that when C_0 is regular as well as coherent and C_t is obtained from C_0 by Jeffrey Conditionalization on a partition, C_t obtained so is also regular as well as coherent. Nissan-Rozen would accept that if the Principal Principle is preserved under Jeffrey Conditionalization, it should hold that:

> **PRESERVATION**: When an agent's rational initial credence function C_0 obeys PP_0 and C_t is updated from C_0 by Jeffrey Conditionalization, C_t obeys PP_t.

In what follows, I will provide an argument against PRESERVATION, and point out a problem regarding the argument. The argument is very similar to Nissan-Rozen's original argument and so the problem will impugn the plausibility of the original argument.

Before providing the argument, briefly consider Jeffrey Conditionalization. When a course of experience directly changes an agent's credence in E from $C_0(E)$ to $C_t(E)(<1)$ and nothing else, Jeffrey Conditionalization requires that:

JC-on-$\{E,\neg E\}$: For any proposition A,
$C_t(A)=C_t(E)C_0(A|E)+C_t(\neg E)C_0(A|\neg E)$,
when $0<C_0(E)<1$.

It is noteworthy here that JC-on-$\{E,\neg E\}$ is equivalent to:

Rigidity: For any proposition A,
$C_t(A|E)=C_0(A|E)$ and $C_t(A|\neg E)=C_0(A|\neg E)$
when $0<C_0(E)<1$.

Now, let me argue against PRESERVATION.

First, suppose that your credence in E is directly changed from $C_0(E)$ to $C_t(E)(<1)$ and nothing else. Here C_0 is your rational initial credence function.[5] Then, we have that:

(1) $C_0(E) \neq C_t(E)<1.$

Suppose also that C_0 obeys PP_0. Then, it holds that:

(2) $C_0(E|TH) = ch_{TH}(E).$

Suppose even that C_t is updated from C_0 by Jeffrey Conditionalization on {E,¬E}. That is, assume JC-on-{E,¬E}. Then, Rigidity implies that:

(3) $C_0(TH|E) = C_t(TH|E);$

(4) $C_0(TH|\neg E) = C_t(TH|\neg E)$

Finally, let's suppose, for *reductio*, that C_t obeys PP_t. Then, it holds that:

(5) $C_t(E|TH) = ch_{TH}(E).$

It is not difficult to show that (2), (3), (4) and (5) jointly imply that $C_0(E)=C_t(E)$, which contradicts (1).[6] So, (1)-(5) jointly imply a

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[5] Note that the rational initial credence function is assumed to be regular. Thus, it holds that $0<C_0(E)<1$ and $0<C_0(TH)<1$. Moreover, it also holds that $0<C_t(TH)<1$. This is because C_t is updated from C_0 by Jeffrey conditionalization on {E,¬E}.

contradiction. To sum up, the above argument shows that PP_0, PP_t and JC-on-{E,¬E} jointly yield a contradiction. Hence, it seems that we should conclude that when PP_0 and JC-on-{E,¬E} holds, PP_t should be violated—that is, PRESERVATION is not the case. However, this conclusion is somewhat hasty. This argument has a serious problem, unfortunately.

3. How the Principal Principle Constrains Posterior Credences

At first glance, Lewis's original Principal Principle *itself* seems to stay silent on the relation between chances and an agent's posterior credences obtained after the agent undergoes a course of experience. However, this silence is merely apparent. Indeed, Lewis's original Principal Principle *itself*, i.e., PP_0^{-} or PP_0, constrains an agent's posterior credences functions. To see this, let me explain how PP_0 *itself* constrains the posterior credence functions.

Suppose that an agent's credence function is updated from C_0 to C_t after the agent undergoes some courses of experience at time t. Here

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6) According to probability calculus, it holds that:

$$C_0(E|TH)=\frac{[C_0(E)/C_0(\neg E)]C_0(TH|E)}{[C_0(E)/C_0(\neg E)]C_0(TH|E)+C_0(TH|\neg E)}$$

$$C_t(E|TH)=\frac{[C_t(E)/C_t(\neg E)]C_t(TH|E)}{[Ct(E)/Ct(\neg E)]Ct(TH|E)+Ct(TH|\neg E)}.$$

Then, it follows from (2)-(5) that:

$$\frac{[C_0(E)/C_0(\neg E)]C_t(TH|E)}{[C_0(E)/C_0(\neg E)]C_t(TH|E)+C_t(TH|\neg E)}=\frac{[C_t(E)/C_t(\neg E)]C_t(TH|E)}{[C_t(E)/C_t(\neg E)]C_t(TH|E)+C_t(TH|\neg E)}$$

We obtain from this equation that:

$$\frac{C_t(E)/C_t(\neg E)}{C_0(E)/C_0(\neg E)}=1.$$

In other words, $C_t(E)=C_0(E)$, which contradicts (1).

C_0 is assumed to be probabilistically coherent and C_t is updated from C_0 by a coherent belief updating rule. By ‘a coherent belief updating rule’, I means a belief updating rule under which probabilistic coherence is preserved. (For example, (Jeffrey) Conditionalization is a coherent belief updating rule.) For the sake of simplicity, consider a finite outcome space $\Omega=\{w_1, \cdots, w_n\}$ that is a set whose members are mutually exclusive and collectively exhaustive. Suppose that C_0 and C_t are measured on this space. Any proposition to which the agent assigns a credence is a disjunction of w_is. Thus, w_i logically implies one of A and ¬A, for any proposition A. Note that the logical relation between w_i and a proposition A remains the same regardless of the agent’s credence function. So, we have that: For any A and w_i,

(6) $C_0(A|w_i) = C_t(A|w_i)$, where $C_0(w_i)$, $C_t(w_i)>0$.

This equation shows how the initial credence function should be related to the posterior one. I should emphasize here that the relation in question does not depend on any particular coherent belief updating rule—e.g., (Jeffrey) Conditionalization. If C_0 is probabilistically coherent and C_t is updated from C_0 by any coherent belief updating rule, then the above equation should hold for C_0 and C_t.

With the help of (6) and probability calculus, then, we have that:[7]

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7) Let W be the set $\{w_i \in \Omega: C_t(w_i)>0\}$. Then, (6) and probability calculus imply that:

(*)
$$\begin{aligned} C_t(A) &= \sum_{w_i \in W} C_t(w_i) C_t(A|w_i) \\ &= \sum_{w_i \in W} C_t(w_i) C_0(A|w_i) = \sum_{w_i \in W} \pi(w_i) C_0(Aw_i), \end{aligned}$$

where $\pi(w_i)=C_t(w_i)/C_0(w_i)$. Note that:

For any A,

(7) $C_t(A) = \sum_{w_i \in \Omega} \pi(w_i) C_0(Aw_i)$, where $\pi(w_i) = C_t(w_i)/C_0(w_i)$.

Now, suppose also that C_0 obeys PP_0. Thus, we have that: For any A,

(8) $C_0(A|TH) = ch_{TH}(A)$.

Using (7) and (8), then, we can formulate a generalized version of the Principal Principle:[8)]

GPP: For any time t and any proposition A,

$$C_t(A|TH) = \frac{\sum_{w_i \in \Omega} \pi(w_i) ch_{TH}(Aw_i)}{\sum_{w_i \in \Omega} \pi(w_i) ch_{TH}(w_i)}$$

where T is the complete theory of chance at a world, H

$$\sum_{\omega_i \in \Omega} \pi(w_i) C_0(Aw_i) = \sum_{w_i \in W} \pi(w_i) C_0(Aw_i) + \sum_{w_i \in \Omega - W} \pi(w_i) C_0(Aw_i).$$

Here, $\Omega - W = \{w_i \in \Omega: C_t(w_i) = 0\}$. Thus, for any $w_i \in \Omega - W$, $\pi(w_i) = C_t(\omega_i) / C_0(w_i) = 0$. Hence, it holds that:

(**) $$\sum_{w_i \in W} \pi(w_i) C_0(Aw_i) = \sum_{w_i \in \Omega} \pi(w_i) C_0(Aw_i).$$

Therefore, Equation (7) follows from (*) and (**).

8) It follows from (7) and (8) that:

$$C_t(A|TH) = \frac{C_t(ATH)}{C_t(TH)} = \frac{\sum_{w_i \in \Omega} \pi(w_i) C_0(ATHw_i)}{\sum_{w_i \in \Omega} \pi(w_i) C_0(THw_i)}$$
$$= \frac{\sum_{w_i \in \Omega} \pi(w_i) C_0(Aw_i|TH)}{\sum_{w_i \in \Omega} \pi(w_i) C_0(w_i|TH)} = \frac{\sum_{w_i \in \Omega} \pi(w_i) ch_{TH}(Aw_i)}{\sum_{w_i \in \Omega} \pi(w_i) ch_{TH}(w_i)}.$$

is the complete history up to a time at the world and ch_{TH} is a chance distribution that is obtained at the time and the world.

Note that PP_0, with probability calculus, implies GPP. Nothing about the way of updating credence functions is assumed except that the associate belief updating rule is coherent. In particular, it is not assumed that the agent updates her credences using (Jeffrey) Conditionalization. Nothing about the posterior credence function C_t is assumed except that the function is probabilistically coherent. In particular, nothing about regularity is assumed. Thus, we should conclude that if your rational initial credence function obeys PP_0, your coherent posterior credence function cannot help obeying GPP whether your coherent posterior credence function is obtained by (Jeffrey) Conditionalization or not, and whether your posterior credence function is regular or not.

Now, revisit Nissan-Rozen's argument—in particular, my revised version of Nissan-Rozen's original argument. The argument shows that PP_0, PP_t and JC-on-$\{E,\neg E\}$ jointly yield a contradiction. Note first that GPP is incompatible with PP_t when there is a chance function ch_{TH} such that for any $w_k \in \Omega$, $ch_{TH}(w_k)>0$. Here is the proof:

Proof.

Suppose first that for any $w_k \in \Omega$, $ch_{TH}(w_k)>0$. Now, suppose, for *reductio*, that both GPP and PP_t hold. That is, assume that for any A,

$$C_t(A|TH) = ch_{TH}(A)$$

$$C_t(A|TH) = \frac{\sum_{w_i \in \Omega} \pi(w_i) ch_{TH}(Aw_i)}{\sum_{w_i \in \Omega} \pi(w_i) ch_{TH}(w_i)}.$$

Then, it follows that: for any $w_k \in \Omega$,

$$ch_{TH}(w_k) = \frac{\sum_{w_i \in \Omega} \pi(w_i) ch_{TH}(w_k w_i)}{\sum_{w_i \in \Omega} \pi(w_i) ch_{TH}(w_i)}.$$

Note that, for any $w_k \in \Omega$,

$$\sum_{w_i \in \Omega} \pi(w_i) ch_{TH}(w_k w_i) = \pi(w_k) ch_{TH}(w_k).$$

This is because, for any i and j such that $i \neq j$, w_i and w_j are incompatible with each other. Recall that, for any $w_k \in \Omega$, $ch_{TH}(w_k) > 0$. Then, it follows from the above equations that, for any $w_k \in \Omega$,

$$\pi(w_k) = \sum_{w_i \in \Omega} \pi(w_i) ch_{TH}(w_i).$$

Thus, it should hold that $\pi(w_1) = \cdots = \pi(w_n)$, which cannot be the case. To demonstrate this, let a be a number such that $\pi(w_1) = \cdots = \pi(w_n) = \alpha$ Then, we have that for any w_i,

$$C_t(w_i) = \alpha C_0(w_i).$$

So, it holds that

$$\sum_{w_i \in \Omega} C_t(w_i) = \sum_{w_i \in \Omega} \alpha C_0(w_i)$$

Finally, we have that $\alpha = 1$ since

$$\sum_{w_i \in \Omega} C_t(w_i) = \sum_{w_i \in \Omega} C_0(w_i) = 1.$$

However, this result contradicts the assumption that C_0 is changed to C_t. Hence, we should conclude that GPP is incompatible with PP_t. Done.

On the other hand, it was shown that PP_0 implies GPP. Therefore, PP_t is incompatible with PP_0 when there is a chance function ch_{TH}

such that for any $w_k \in \Omega$, $ch_{TH}(w_k)>0$. As a result, we may conclude that the contradiction in Nissan-Rozen's argument has little to do with JC-on-{E,¬E}—i.e., Jeffrey Conditionalization. Rather, the contradiction in question seems to be due to the incompatibility between PP_t and PP_0. Admittedly, this result is not conclusive since it was assumed that there is a chance function ch_{TH} such that for any $w_k \in \Omega$, $ch_{TH}(w_k)>0$. However, this result seems to be sufficient to disclose the implausibility of Nissan-Rozen's argument. This implausibility is due to a wrong formulation of the relationship between the Principal Principle and posterior credences.

Before finishing my discussion, I would like to emphasize some interesting features of GPP. First, consider a relationship between PP_0 and GPP. As was shown, PP_0 implies GPP. What about the converse? Interestingly, PP_0 is a special case of GPP—that is, GPP implies PP. More exactly, when t=0, GPP is equivalent to PP_0. Note that when t=0, $\pi(w_i)=1$ for any $w_i \in \Omega$. Suppose that t=0. Then, GPP, with probability calculus, implies that:

$$C_0(A|TH) = \frac{\sum_{w_i \in \Omega} ch_{TH}(Aw_i)}{\sum_{w_i \in \Omega} ch_{TH}(w_i)} = ch_{TH}(A).$$

That is, when t=0, GPP is equivalent to PP_0; hence, GPP implies PP_0. Recall that PP_0 implies GPP. As a result, we can conclude that GPP is equivalent to PP_0.

Second, consider the relationship between GPP and (Jeffrey) Conditionalization. If the Principal Principle is preserved under a coherent belief updating rule **R**, it should hold that:

PRESERVATION*: When an agent's rational initial credence function C_0 obeys PP_0 and C_t is updated from C_0 by the coherent belief updating rule **R**, C_t obeys GPP.

I have proved that, when an agent's rational initial credence function C_0 obeys PP_0 and C_t is updated from C_0 by a coherent belief updating rule, C_t obeys GPP. This result shows that PRESERVATION* holds for any coherent belief updating rule. Thus, we can conclude that the Principal Principle is preserved under (Jeffrey) Conditionalization. (Note that (Jeffrey) Conditionalization is a coherent belief updating rule.)

4. Conclusion

In this paper, I have shown that, contrary to what many philosophers of chance have thought, Lewis's original Principal Principle itself does not stay silent on how our posterior credences should be related to chances. That is, Lewis's principle itself requires that an agent's posterior credences should obey GPP. Furthermore, I have proved, with the help of this result, that Nissan-Rozen's argument fails to show that the Principal Principle is not preserved under Jeffrey Conditionalization. Indeed, the Principal Principle is preserved under any coherent belief updating rule. Of course, Jeffrey Conditionalization is such a rule.[9)]

[9)] As is well known, there is theoretical tension between Lewis's Humean account of chance and his original Principal Principle. So, some philosophers have proposed a revised version of the principle (see Lewis (1994), Hall (1994) and Thau (1994) for relevant discussion). Here is one new version of the principle (see Hall 1994):

NP_0: For any proposition A, $C_0(A|TH) = ch_{TH}(A|T)$, where T is the complete theory of chance at a world, H is the complete history up to a time at the world and ch_{TH} is a chance distribution that is obtained at the time and the world.

Of course, NP_0 can be generalized in a similar way to GPP. Here is such a generalization:

GNP: For any time t and any proposition A,

$$C_t(A|TH) = \frac{\sum_{w_i} \pi(w_i) ch(Aw_i|T)}{\sum_{w_i} \pi(w_i) ch(w_i|T)}$$

where T is the complete theory of chance at a world,
H is the complete history up to a time at the world and ch_{TH} is a chance distribution that is obtained at the time and the world.
It is not difficult to ascertain that NP_0 is equivalent to GNP.

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주요 원리와 사후 신념도

박 일 호

많은 확률철학자들은 루이스의 주요 원리가 우리의 사후 신념도에 대해서는 침묵하고 있다고 생각한다. 즉 주요 원리는 아무런 정보도 가지고 있지 않은 초기 신념도에 대한 것뿐이라는 것이다. 하지만 본 논문은 이런 일반적인 생각과 반대로 루이스의 주요 원리 그 자체가 이미 우리의 사후 신념도가 물리적 확률(chance)와 어떻게 결합되어야 하는지 말해주고 있다고 주장할 것이다. 이와 더불어, 주요 원리는 제프리 조건화를 통해서 보존되지 않는다는 니산-로젠의 최근 주장을 비판적으로 검토할 것이다. 결국, 본 논문을 통해 주요 원리가 제프리 조건화를 통해서 보존된다는 것이 논증될 것이다.

주요어: 주요 원리, 초기 신념도, 사후 신념도, 제프리 조건화