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A Defense of Imprecise Credences: Objections to Topey[†]

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When an agent has only unspecific evidence regarding P, what is an epistemically rational response to that evidence? In his “A Defense of Imprecise Credences in Inference and Decision Making”, James Joyce asserts that the response should be to have an imprecise credence in P. Recently, in his “Coin Flips, Credences and the Reflection Principle”, Brett Topey, considering Roger White's coin puzzle, suggests two interesting arguments against Joyce's view. In this paper, however, I show that each one has a flaw, and thus that no proponents of imprecise credences would be persuaded by his arguments.

【Keywords】imprecise credences, Roger White's coin puzzle, Reflection principle, unanimity rule for the imprecise model

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1. Introduction

When an agent has only *unspecific* evidence regarding *P*, what is her epistemically rational response to that evidence? Joyce(2010) asserts that the response should be to have an *imprecise* credence in *P*. Topey(2012), however, considering White(2010)'s coin puzzle, suggests two interesting arguments against Joyce's view. In sections 3 and 4, however, we will see that each one has a flaw, and thus that no proponents of imprecise credences would be persuaded by his arguments.

Before presenting my discussion, let me review several assumptions regarding the way we model an agent's belief states. We will assume that, following Joyce, an agent's belief state can be represented by a set (a representor) $\boldsymbol{C}$ of precise credence functions c (subjective probability functions). For instance, when the agent has a precise credence 1/2 in *P*, $\boldsymbol{C}$ consists of a set of credence functions, all of which give *P* the value 1/2. When she has an imprecise credence in *P*, the members of $\boldsymbol{C}$ disagree about the value for *P*. For convenience, I will also use an interval to represent the range of values that the members of $\boldsymbol{C}$ give to *P*.[1] We will also assume that the representor of an ideal agent is updated by (Bayesian) conditionalization on each of its members. That is, if a rational agent in credal state $\boldsymbol{C}$ learns *E* with certainty and nothing else, then her post-learning credal state will be $\boldsymbol{C_E} = \{c(\bullet \mid E): c \in \boldsymbol{C}\}$. Finally, I will accept the following unanimity rule for the set of precise credence functions as Joyce does: If and only if all credence functions in the representor agree about some matter, this reflects a determinate fact about the agent's belief. For instance, if $c(Y) > c(X)$

1) Here it is assumed that the following constraint holds in the imprecise credence model: Convexity: If c_i, $c_j \in \boldsymbol{C}$, then $\alpha c_i + (1\text{-}a)c_j \in \boldsymbol{C}$, $a \in (0, 1)$.

for every $c \in \boldsymbol{C}$, it shows the fact that the agent regards Y as more likely than X. But if $c(Y) > c(X)$ for some $c \in \boldsymbol{C}$ and $c(Y) \leq c(X)$ for the rest, then it is indeterminate whether she regards Y as more likely than X.

Now let's play the coin puzzle!

2. The Coin Puzzle, and a Mistake?

Topey has paraphrased White's coin puzzle as follows:

> "Jack has a coin that Mark knows to be fair. In addition, Mark has no idea whether *P* but knows that Jack knows whether *P*. Jack painted the coin so that Mark can't see which side is heads and which side is tails, then writes '*P*' on one side and '~*P*' on the other, explaining to Mark that he has placed whichever is true on the heads side and that he will soon toss the coin so that Mark can see how it lands. Jack tosses the coin, and Mark observes that it has landed with the side marked '*P*' facing up."[2)]

Let $\boldsymbol{C}$ refer to Mark's set of initial rational credence functions, $\boldsymbol{C}_+$ refer to Mark's set of rational credence functions after learning the coin landed '*P*' side up, and H and T refer to a proposition that the coin lands heads and a proposition that the coin lands tails, respectively. Topey lists the following assertions about the coin puzzle:[3)]

(1) $\boldsymbol{C}(P) = [0, 1]$
(2) $\boldsymbol{C}(H) = 0.5$

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2) Topey (2012), pp. 478-79.
3) Ibid., p. 481.

(3) $\boldsymbol{C}_{+}(P) = \boldsymbol{C}_{+}(H)$

(4) $\boldsymbol{C}_{+}(P) = \boldsymbol{C}(P)$

(5) $\boldsymbol{C}_{+}(H) = \boldsymbol{C}(H)$

(1) – (5) are jointly inconsistent. White and Topey reject (1). But Joyce rejects (5). According to Topey, (2) and (3) are uncontroversial, and (4) is 'quite reasonable'. If we accept Lewis (1980)'s Principal principle, according to which, one's credences in propositions about objective chances should constrain her credences in other propositions[4], (2) is plainly true, because Mark knows that the coin is fair and so the chance of landing heads is ½.

Mark does not learn anything new about *P* before or after flipping the coin, and so it is quite reasonable that his credence in *P* should not change, as (4) says.

What about (3)? At this point, some may think that Topey makes a mistake. Why? To see this, following Topey, let Mark's set of after-the-toss credence functions be $\boldsymbol{C}_{+}$. Topey defines $\boldsymbol{C}_{+}$ as follows: $\boldsymbol{C}_{+}(X) = \{c(X \mid P \equiv H): c \in \boldsymbol{C}\}$. Note that Topey thinks that Mark updates $\boldsymbol{C}$ by conditionalizing on $P \equiv H$. We can easily prove that $c(P \mid P \equiv H) = c(H \mid P \equiv H)$ for any c in $\boldsymbol{C}$.[5] Therefore, based on Topey's definition of $\boldsymbol{C}_{+}$, (3) is true. However, some may wonder whether Topey's definition of $\boldsymbol{C}_{+}$ is based on a correct application of conditionalization. In the coin puzzle, according to Topey, "when Mark sees the coin land with the '*P*' side up, he learns that $P \equiv H$."

4) More formally, if *E* is a rational agent's total (admissible) evidence at t, for any proposition *X*, $c(X \mid E \;\&\; \text{chance}_t(X) = x) = x$, given $c(E \;\&\; \text{chance}_t(X) = x) > 0$, where c is her (initial) credence function and $\text{chance}_t(X)$ is an objective chance of *X* at t.

5) Note that $c(P \mid P \equiv H) = c(P \;\&\; (P \equiv H)) / c(P \equiv H) = c(H \;\&\; (P \equiv H)) / c(P \equiv H) = c(H \mid P \equiv H)$ for any c in $\boldsymbol{C}$.

This appears to be so because ‘$P \equiv H$’ follows from ‘the coin lands with the ‘*P*’ side up’ plus knowledge of setup that the fair coin is painted and newly labeled with *P* on one side and ~*P* on another side, with the true placed on the heads side. Assuming knowledge of setup, would ‘the coin lands with the ‘*P*’ side up’ be equivalent to ‘$P \equiv H$’? At first glance, it seems not. The coin landing with the ‘*P*’ side up does not seem to follow from ‘$P \equiv H$’ plus the knowledge of setup. ‘$P \equiv H$’ does not seem to tell us what came up. That is, given the knowledge of setup, ‘the coin lands with the ‘*P*’ side up’ implies ‘$P \equiv H$’ but the reverse does not seem to hold. Therefore, assuming knowledge of setup, ‘the coin lands with the ‘*P*’ side up’ seems to contain more information than ‘$P \equiv H$’ does.

As well known, conditionalization requires that credence updaters should conditionalize on the proposition that captures *all* that they *newly* learn with certainty. As I mentioned above, at least for Mark, assuming the knowledge of setup, ‘the coin lands with the ‘*P*’ side up’ appears to contain more information than ‘$P \equiv H$’ does. Therefore, Mark seems to have to conditionalize on ‘the coin lands with the ‘*P*’ side up’, not on ‘$P \equiv H$’.

Let P^* be the proposition that the coin lands with the ‘*P*’ side up. Assuming knowledge of setup, if P^* contains more information than ‘$P \equiv H$’ does, according to conditionalization, then, it should hold that: For an arbitrary proposition *X*, $\boldsymbol{C}_+(X) = \boldsymbol{C}(X \mid P^*) = \{c(X \mid P^*): c \in \boldsymbol{C}\}$. *H* and P^* appear to be independent of each other relative to any credence function that is a member of $\boldsymbol{C}$. The coin landing ‘*P*’ side up seems not give any new information at all about whether the coin lands heads or tails to Mark. Then, $\boldsymbol{C}(\text{heads} \mid P^*) = 1/2$. Moreover, it appears that *P* and P^* are independent of each other relative to any credence function in $\boldsymbol{C}$. (How would the fact that the coin landed ‘*P*’ sides up give any new information about whether *P*?)

If so, $\boldsymbol{C}(P \mid P^*) = [0, 1]$. In case that P^* contains more information than '$P \equiv$heads' does, after the application of conditionalization, $\boldsymbol{C}_+(P) = \boldsymbol{C}(P \mid P^*) = [0, 1] \neq \boldsymbol{C}_+(\text{heads}) = \boldsymbol{C}(\text{heads} \mid P^*) = 1/2$. Therefore, if P^* contains more information than '$P \equiv H$' does, (3) would be false.

However, regarding (3), Topey does not make such a mistake, because, as opposed to what it appears to be, assuming the knowledge of setup that the fair coin is painted and newly labeled with P on one side and $\sim P$ on another side, and Jack has placed whichever is true on the heads side, the proposition that the coin lands with the 'P' side up (P^*) is equivalent to $P \equiv H$. Here is the proof of the equivalence:

From Mark's knowledge of setup, it follows that he knows the followings:

(i) P is true if and only if 'P' is painted on the heads side of the coin.
(ii) The coin lands either heads or tails.
(iii) 'P' is painted on either the heads side of the coin or the tails side.

Now, to prove the equivalence, let me start by proving that, given (i)-(iii), P^* implies $P \equiv H$. Let's assume that the coin lands with the 'P' side up (P^*). Then, by (i), it follows that if P is true, then the coin lands heads, and that if P is false, then the coin lands tails. Thus, given (i), P^* implies $P \equiv H$.

Next, we can show that $P \equiv H$ implies P^*. To see this, assume that P is true if and only if the coin lands heads ($P \equiv H$). Then, by (i), it follows that the coin lands heads if and only if 'P' is painted on the heads side of the coin. That is, from $P \equiv H$ and (i), it follows that either the coin does land heads and ‘*P*’ is painted on the heads side of the coin or the coin does not land heads and ‘*P*’ is not painted on the heads side of the coin. Then, by (ii) and (iii), we have: either the coin lands heads and ‘*P*’ is painted on the heads side of the coin or the coin lands tails and ‘*P*’ is painted on the tails side of the coin. Then it immediately follows from this disjunction that the coin lands with the ‘*P*’ side up. Thus, given (i)-(iii), $P \equiv H$ implies P^*.

To sum up, on the assumption of knowledge of setup, then by (i)-(iii), P^* is equivalent to $P \equiv H$. Thus, it holds that $c(P \mid P \equiv H) = c(H \mid P \equiv H) = c(P \mid P^*) = c(H \mid P^*)$ for any c in $\boldsymbol{C}$. That is, given knowledge of setup, (3) is also definitely true.

Then, as Topey claims, should we reject (1)? No, we are not required to reject (1), as I will explain below.

3. Psychological Differences and Reflection’s Applicability

Topey suggests two objections to the orthodox proponents of imprecise credences. However, each one has a flaw, and thus no orthodox proponents of imprecise credences would be persuaded by his arguments.

His first argument is based on the Reflection principle, formulated and named by van Fraassen(1984). The Reflection principle is a constraint on a rational relationship between an agent’s current and future opinions. It states that one’s current self should epistemically defer to one’s future self. The Reflection principle may be understood in many ways. In discussing the reflective idea, many seem to assume the following principle:

Special Reflection principle (SRP): For any proposition *X*,

> when you know that you will come to have a *precise* credence in X at future time t, you should currently have that *precise* credence in X.

Note that SRP applies only to cases where one knows that she will come to have a *precise* credence in X at t. In order to apply the idea of SRP to the case where one knows that she will come to have an *imprecise* credence in X at t, following White[6], Topey (implicitly) assumes the following generalized version of SRP:

> **Generalized Special Reflection principle (GSRP)**: For any proposition X, when you know that you will come to have a doxastic attitude to X at future time t, you should currently have that doxastic attitude to X.

Note that when a doxastic attitude to X at t is precise, GSRP is same with SRP, but even if a doxastic attitude to X at t is imprecise, in contrast to SRP, we can still apply GSRP to the case where one knows that she will come to have an *imprecise* belief state in X at t.

Topey[7] follows White[8] in suggesting the following objection to the proponents of imprecise credences:

(I) $\boldsymbol{C}_{+}(H) = \boldsymbol{C}(H \mid H \equiv P)$ or $\boldsymbol{C}_{+}(H) = \boldsymbol{C}(H \mid H \equiv {\sim}P)$ but not both.

(II) $\boldsymbol{C}(H \mid H \equiv P) = \boldsymbol{C}(H \mid H \equiv {\sim}P) = [0, 1]$ which entails $\boldsymbol{C}_{+}(H) = [0, 1]$.

(III) Therefore, since the agent is sure that $\boldsymbol{C}_{+}(H) = [0, 1]$, applying GSRP to $\boldsymbol{C}(H)$ and $\boldsymbol{C}_{+}(H)$ yields $\boldsymbol{C}(H) = [0, 1]$, which is contradictory to $\boldsymbol{C}(H) = 1/2$.

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6) White (2010), p. 178.
7) Topey (2012), p. 482.
8) White (2010), p. 178.

It is clear that GSRP can be applied to the coin puzzle when $\boldsymbol{C}_{H\equiv P}(H)$ and $\boldsymbol{C}_{H\equiv\sim P}(H)$ represent the *same* psychological state (i.e., the *same* imprecise or indeterminate future belief state).[9] However, as Joyce points out, even though $\boldsymbol{C}(H \mid H\equiv P)$ and $\boldsymbol{C}(H \mid H\equiv\sim P)$ have the same range of credences for H, $\boldsymbol{C}_{H\equiv P}(H)$ and $\boldsymbol{C}_{H\equiv\sim P}(H)$ represent different psychological states. For instance, "from [Mark's] prior perspective, [his] belief about [*H*] upon learning [$H\equiv P$] are antithetical to [his] beliefs about [*H*] upon learning [$H\equiv\sim P$]. Learning [$H\equiv P$] puts [him] in a position in which [his] uncertainty about [*H*] versus [*T*] is uncertainty about [*H*&*P*] versus [*T*&~*P*]. Learning [$H\equiv\sim P$] puts [him] in a position in which [his] uncertainty about [*H*] versus [*T*] is uncertainty about [*H*&~*P*] versus [*T*&*P*]."[10] To see this, note that, in the coin puzzle, learning that Jack has placed *P* on the heads side puts Mark in a position in which his uncertainty about *H* versus *T* is uncertainty about *H*&*P* versus *T*&~*P*. In contrast, learning that Jack has placed *P* on the tails side puts Mark in a position in which his uncertainty about *H* versus *T* is uncertainty about *H*&~*P* versus *T*&*P*.

Topey also accepts a similar point. He thinks the following equations between conditional credences show the psychological distinction between two possible future belief states:

$$\boldsymbol{C}_{H\equiv P}(H \mid P) = \boldsymbol{C}_{H\equiv P}(P \mid H) = 1;\ \boldsymbol{C}_{H\equiv P}(T \mid \sim P) = \boldsymbol{C}_{H\equiv P}(\sim P \mid T) = 1;$$
$$\boldsymbol{C}_{H\equiv\sim P}(H \mid \sim P) = \boldsymbol{C}_{H\equiv\sim P}(\sim P \mid H) = 1;\ \boldsymbol{C}_{H\equiv\sim P}(T \mid P) = \boldsymbol{C}_{H\equiv\sim P}(P \mid T) = 1.$$

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9) Here $\boldsymbol{C}_{H\equiv P}$ and $\boldsymbol{C}_{H\equiv\sim P}$ refer to Mark's set of credence functions after learning $H\equiv P$ and Mark's set of credence functions after learning $H\equiv\sim P$, respectively.

10) Joyce (2010), p. 303. The brackets are mine.

However, Topey asserts that the psychological differences that the conditional credences show cannot prevent us applying the Reflection principle to the coin puzzle, because "the differences are not differences in [$\boldsymbol{C}_{+}(H)$] and so are irrelevant to the question at hand ."[11] That is, "the question of the value of [$\boldsymbol{C}_{+}(H \mid P)$ or $\boldsymbol{C}_{+}(H \mid {\sim}P)$] is simply separate from the question of the value of [$\boldsymbol{C}_{+}(H)$]":

> "The point is clear when the questions are asked in informal terms: "Tomorrow, how confident Mark be that [H]? and Tomorrow, how confident will Mark be that [H] given the assumption that [P]" are simply two different questions, and our ability to apply Reflection based on the answer to the first question has nothing to do with the answer to the second."[12]

Why should we think that 'the values of $\boldsymbol{C}_{+}(H \mid P)$ and $\boldsymbol{C}_{+}(H \mid {\sim}P)$' are irrelevant to 'the value of $\boldsymbol{C}_{+}(H)$'? Suppose that tomorrow, Mark will learn either '$H \equiv P$' or '$H \equiv {\sim}P$' and nothing else. Then, tomorrow, depending on what he learns, Mark's credence in H will be updated into $\boldsymbol{C}_{H \equiv P}(H)$ or $\boldsymbol{C}_{H \equiv {\sim}P}(H)$ by conditionalizing on it. Let's assume that Mark learns '$H \equiv P$'. Then, as Topey points out, $\boldsymbol{C}_{H \equiv P}(H \mid P) = \boldsymbol{C}_{H \equiv P}(P \mid H) = 1$, which entails $\boldsymbol{C}_{H \equiv P}(H) = \boldsymbol{C}_{H \equiv P}(H \& P)$. Doesn't this point show how strongly the values of $\boldsymbol{C}_{+}(H \mid P)$ and $\boldsymbol{C}_{+}(H \mid {\sim}P)$ are relevant to the value of $\boldsymbol{C}_{+}(H)$?

However, regardless of whether it does so or not, Topey's point *misses the target*. Joyce[13] never assert that psychological distinctions should prevent us from applying the Reflection principle to the coin puzzle. That is, Joyce never asserts that the Reflection principle

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11) Here and below I have replace Topey's variables and symbols with the analogous ones from my own presentation.

12) Topey (2012), p. 485.

13) van Fraassen (1995, p. 19) as well.

cannot be applied in cases in which an agent considers several distinct possible future credal states. Before presenting Joyce's view, however, note that Topey's first argument is based on the particular version of Reflection (GSRP). There is another version of the principle as follows:

> **ERP**: For any proposition *X*, your current opinion about *X* must be equal to your current expectation of your future opinions for *X* at later time t. More formally, let *c* and c_t be your current credence function and your credence function at some future time t, respectively. Then for any *X*, you should have $c(X) = \sum_i x_i \times c(c_t(X) = x_i)$.

Note that an agent is able to have a current expectation of her possible future opinions, each of which comes from a distinct psychological state. Joyce advocates that if we understand the Reflection principle in this way, we can apply it to the coin puzzle in which there is psychological difference between $\boldsymbol{C}_{H\equiv P}$ and $\boldsymbol{C}_{H\equiv\sim P}$, regardless of whether they are relevant or irrelevant to 'the value of $\boldsymbol{C}_{+}(H)$', as I will explain.

As Joyce[14] points out, "from ***C***'s perspective, [$\boldsymbol{C}_{H\equiv P}$ and $\boldsymbol{C}_{H\equiv\sim P}$] encode complementary beliefs about [*H*], even though each is maximally imprecise about its probability."[15] That is, all credence functions in ***C*** agree that $c_{H\equiv P}(H) = 1 - c_{H\equiv\sim P}(H)$. So, for any *c* in ***C***, $c_{H\equiv P}(H) + c_{H\equiv\sim P}(H) = 1$. Thus, if $c_{H\equiv P}(H) = r$, $c_{H\equiv\sim P}(H) = 1 - r$ for any $r \in [0, 1]$.[16] And it is assumed that, in the coin puzzle, for any *c* in ***C***, $c(H\equiv P) = c(H\equiv\sim P) = 1/2$. Thus, for any *c* in ***C***, $c(c_{H\equiv}$

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14) Joyce (2010), p. 302.
15) The brackets are mine.
16) Note that a value of *r* is identical to a value that each *c* in ***C*** gives *P*.

$_P(H) = r)) = c(c_{H\equiv\sim P}(H) = 1 - r)$; let it be a. Regarding this point, it is also assumed that Mark is certain that he updates his credences by conditionalization. By our assumption that Mark will learn one biconditional or the other, about which all credence functions agree, a=1/2.

Thus, applying ERP to the coin puzzle, we have that: For any c in $\boldsymbol{C}$,

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\begin{aligned}
c(H) &= c(H \mid c_{H\equiv P}(H) = r) \times c(c_{H\equiv P}(H) = r) \\
&\quad + c(H \mid c_{H\equiv\sim P}(H) = 1 - \mathrm{r}) \times c(c_{H\equiv\sim P}(H) = 1 - \mathrm{r}) \\
&= r \times c(c_{H\equiv P}(H) = r) + (1 - \mathrm{r}) \times c(c_{H\equiv\sim P}(H) = 1 - \mathrm{r}) \\
&= (1/2)\times r + (1/2)\times(1 - \mathrm{r}) \\
&= 1/2.
\end{aligned}
$$

As we can clearly see, Joyce never asserts that psychological differences can prevent us applying the Reflection Principle to the coin puzzle. Moreover, as opposed to (3), the combination of ERP and imprecise credence does not entail $\boldsymbol{C}(H)$ = [0, 1], but $\boldsymbol{C}(H)$ = 1/2. Therefore, Topey's first argument will fail to convince the orthodox proponents of imprecise credences.

Some may claim that Topey is working with the different version of the Reflection principle (GSRP), so the disagreement about which version of the Reflection principle applies to the coin puzzle is merely *verbal*. It may be true. However, it is noteworthy that, given ERP and the unanimity rule for imprecise credences, $\boldsymbol{C}(H)$ = 1/2 follows. And, as already pointed out in section 1, proponents of imprecise credences accept the unanimity rule. Thus, even if the disagreement about the versions of the Reflection principle is merely *verbal*, it seems clear that, on the assumption of ERP, proponents of imprecise credences would *not* be convinced by Topey's first argument.[17)]

Some may further claim that Topey's main claim is that the two possible future credences in *H* (i.e., $\boldsymbol{C}_{H\equiv P}(H)$ and $\boldsymbol{C}_{H\equiv\sim P}(H)$) are in fact *identical*; Topey is attempting to show that Joyce's result is just an artifact of Joyce’s imprecise credence (representor) model and that Mark's doxastic attitude toward *H* is the same in both possible cases. That is, Topey is questioning whether the mechanics of the representor model correspond to Mark's psychology in the right way for this formal result to tell us something about Mark's attitude toward *H*. However, if the two possible future credences in *H* (i.e., $\boldsymbol{C}_{H\equiv P}(H)$ and $\boldsymbol{C}_{H\equiv\sim P}(H)$) are in fact *identical*, how can we explain a definite difference between them? For instance, as already pointed out, learning $H\equiv P$ puts Mark in a position in which his uncertainty about *H* versus *T* is uncertainty about *H*&*P* versus *T*&~*P*. In contrast, learning $H\equiv\sim P$ puts him in a position in which his uncertainty about *H* versus *T* is uncertainty about *H*&~*P* versus *T*&*P*. Such a difference should hold, I think, even if Joyce’s imprecise model is not assumed. Is there any good reason why $\boldsymbol{C}_{H\equiv P}(H)$ and $\boldsymbol{C}_{H\equiv\sim P}(H)$ are in fact *identical* even though Mark will think about *H* versus *T* in a different way, depending which proposition he will learn? To respond this point, I think, Topey provides an argument for the identity of $\boldsymbol{C}_{H\equiv P}(H)$ and $\boldsymbol{C}_{H\equiv\sim P}(H)$. However, it also fails to convince proponents of imprecise credences to accept the identity, as I will explain in the next section.

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[17] However, in section 5, I will briefly consider an open *metaphysical* question that might be a real treat to proponents of imprecise credences.

4. Imprecise Credences and the Unanimity Rule for the Representor

Topey's argument for the identity of $\boldsymbol{C}_{H\equiv P}(H)$ and $\boldsymbol{C}_{H\equiv\sim P}(H)$ can be summarized as follows:[18]

(A) $\boldsymbol{C}_{H\equiv P}(H) \neq \boldsymbol{C}_{H\equiv\sim P}(H)$
(B) $\boldsymbol{C}_{H\equiv P}(H) = \boldsymbol{C}_{H\equiv P}(P)$
(C) $\boldsymbol{C}_{H\equiv\sim P}(H) = \boldsymbol{C}_{H\equiv\sim P}(\sim P)$
(D) $\boldsymbol{C}_{H\equiv P}(P) \neq \boldsymbol{C}_{H\equiv\sim P}(\sim P)$ (from (A), (B) and (C))
(E) $\boldsymbol{C}_{H\equiv P}(P) = \boldsymbol{C}(P)$
(F) $\boldsymbol{C}_{H\equiv\sim P}(\sim P) = \boldsymbol{C}(\sim P)$
(G) $\boldsymbol{C}(P) \neq \boldsymbol{C}(\sim P)$ (from (D), (E), and (F))
(H) Therefore, $\boldsymbol{C}_{H\equiv P}(H) = \boldsymbol{C}_{H\equiv\sim P}(H)$

The argument hinges on the psychological difference between $\boldsymbol{C}_{H\equiv P}(H)$ and $\boldsymbol{C}_{H\equiv\sim P}(H)$.[19] (G) is, according to Topey, absurd, since 'Mark has exactly same evidence for P and for $\sim P$ — that is, none '.[20] Thus, Topey concludes that we should accept the denial of (A) (i.e., $\boldsymbol{C}_{H\equiv P}(H) = \boldsymbol{C}_{H\equiv\sim P}(H)$). This argument is valid and Joyce seems to have accepted (B), (C), (E), and (F).[21] Moreover, if Joyce accepts

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18) Topey (2012), p. 486.

19) Of course, if we just take $\boldsymbol{C}_{H\equiv P}(H)$ and $\boldsymbol{C}_{H\equiv\sim P}(H)$ to be an interval, they must be equal since both are [0, 1]. However, as Joyce points out, the interval itself cannot represent all (epistemic) properties that $\boldsymbol{C}_{H\equiv P}(H)$ and $\boldsymbol{C}_{H\equiv\sim P}(H)$ have. Here I take $\boldsymbol{C}_{H\equiv P}(H)$ and $\boldsymbol{C}_{H\equiv\sim P}(H)$ in a more broad sense. That is, $\boldsymbol{C}_{H\equiv P}(H)$ and $\boldsymbol{C}_{H\equiv\sim P}(H)$ express something else such as a complementary relationship between them besides denoting the interval.

20) Ibid.

21) As was shown, we can easily prove (B) and (C). And Joyce thinks that

(A), he also has to accept (D), and (G) follows from the premises. However, Joyce needs not accept (A).[22] Why?

First, note that there are representors $\boldsymbol{C}$ and $\boldsymbol{C}_+$ ($\boldsymbol{C}_{H\equiv P}$ or $\boldsymbol{C}_{H\equiv \sim P}$), and credence functions c and c_+ ($c_{H\equiv P}$ or $c_{H\equiv \sim P}$) belonging to them, respectively, in Joyce's imprecise credence model. In order for Topey's second argument to work, on the assumption of Joyce's imprecise credence model, it should apply to the representor, i.e., all credence functions in the representor, because, as Joyce points out, "Facts about the person's opinions correspond to properties common to all the credence functions in her credal state".[23] For instance, if $c(Y) > c(X)$ for every c in $\boldsymbol{C}$, then the agent has the opinion that she regards Y as more likely than X. But if $c(Y) > c(X)$ for some $c \in \boldsymbol{C}$ and $c(Y) \leq c(X)$ for the rest, then it is *indeterminate* whether she regards Y as more likely than X. To put it another way, in order to make a claim about an agent's *determinate* opinions, the unanimity rule for all members of representor should hold. Even if, for all credence functions except only one in $\boldsymbol{C}$, $c(Y) > c(X)$, that does not show that she regards Y as more likely than X, since there is one

learning the correlations would not give any new information at all about whether P or $\sim P$ to Mark. Thus, Joyce would accept (E) and (F).

22) As far as I know, Joyce himself does not address Topey's arguments against imprecise credences. He clearly points out that from $\boldsymbol{C}$'s perspective, $\boldsymbol{C}_{H\equiv P}(H)$ and $\boldsymbol{C}_{H\equiv \sim P}(H)$ represent different psychological states. However, I do not think that Joyce would take (A) to express the psychological difference between them in a proper way. He emphasizes that a complementary relation between $\boldsymbol{C}_{H\equiv P}(H)$ and $\boldsymbol{C}_{H\equiv \sim P}(H)$ (i.e., for every $c \in \boldsymbol{C}$, $c_{H\equiv P}(H) = 1 - c_{H\equiv \sim P}(H)$) does express such psychological difference between $\boldsymbol{C}_{H\equiv P}(H)$ and $\boldsymbol{C}_{H\equiv \sim P}(H)$. And, as I explain below, the complementary relation between $\boldsymbol{C}_{H\equiv P}(H)$ and $\boldsymbol{C}_{H\equiv \sim P}(H)$ is totally different from (A).

23) Joyce (2010), p. 287.

credence function that does not share this property.

Thus, assuming the imprecise credence model in which the unanimity rule holds, in order for (A) to be a (determinate) fact about Mark's credal state, it should hold that for all credence functions $c \in \boldsymbol{C}$, $c_{H\equiv P}(H) \neq c_{H\equiv \sim P}(H)$. But it does not hold. Why? It is true that $\boldsymbol{C}_{H\equiv P}(H)$ and $\boldsymbol{C}_{H\equiv \sim P}(H)$ represent *distinct* future psychological states, respectively. However, it is noteworthy that (A) does *not* correctly express their difference. As I mentioned before, they have a complementary relation to each other as follows: For every $c \in \boldsymbol{C}$, $c_{H\equiv P}(H) = 1 - c_{H\equiv \sim P}(H)$. That is, as Joyce points out, one is a mirror image of another. This does not entail that, for every $c \in \boldsymbol{C}$, $c_{H\equiv P}(H) \neq c_{H\equiv \sim P}(H)$. There is a credence function c of $\boldsymbol{C}$ such that $c_{H\equiv P}(H) = c_{H\equiv \sim P}(H) = 1/2$. That is, the complementary relation between $\boldsymbol{C}_{H\equiv P}(H)$ and $\boldsymbol{C}_{H\equiv \sim P}(H)$ (i.e., $\boldsymbol{C}_{H\equiv P}(H) = 1 - \boldsymbol{C}_{H\equiv \sim P}(H)$) shows a determinate fact about Mark's credal state, which reveals how $\boldsymbol{C}_{H\equiv P}(H)$ and $\boldsymbol{C}_{H\equiv \sim P}(H)$ are different to each other, but the inequality between $\boldsymbol{C}_{H\equiv P}(H)$ and $\boldsymbol{C}_{H\equiv \sim P}(H)$ (i.e., $\boldsymbol{C}_{H\equiv P}(H) \neq \boldsymbol{C}_{H\equiv \sim P}(H)$) does not. Therefore, it is not true that for all $c \in \boldsymbol{C}$, $c(P) \neq c(\sim P)$ follows from the premises. That is, given the imprecise credence model in which the unanimity rule holds, we cannot conclude that Mark has the *determinate* opinion that the probability of P and the probability of $\sim P$ are unequal. Thus, when applied to the representor (i.e., all credence functions in the representor), Topey's second argument is unsound because premise (A) is not true, i.e., there is one credence function such that $c_{H\equiv P}(H) = c_{H\equiv \sim P}(H) = 1/2$. Topey's argument fails to convince Joyceans to accept (A) and so (G).

It is noteworthy that we cannot conclude the denial of (G) either, for the same reason. We can only conclude that Mark has *indeterminate* opinions about whether the probability of P and the probability of ~*P* are equal or not.[24] This conclusion, I think, is plausible since 'no evidence for *P* and for ~*P*' would not exclude the epistemic possibility $c(P) \neq c(\sim P)$ nor the epistemic possibility $c(P) = c(\sim P)$ for some $c \in \boldsymbol{C}$. Why? One of the central motivations for imprecise credences is to reflect our ignorance when we have only *unspecific* evidence. When we know nothing specific about *P* (and about ~*P*), it seems rational for us to open the epistemic possibility of any credence in *P* (or ~*P*) that is compatible with that unspecific evidence. If $\boldsymbol{C}(P) = \boldsymbol{C}(\sim P)$ holds, however, we have to exclude the epistemic possibility of an inequality between a precise credence in *P* and a precise credence in ~*P*, which is clearly compatible with the ignorance. Thus, $\boldsymbol{C}(P) = \boldsymbol{C}(\sim P)$ cannot properly reflect the ignorance in question.

5. Conclusion

I have argued that Topey's first argument against the imprecise model fails, if the Reflection pinciple is understood in a less restrictive form (ERP). Further, given the imprecise credence model in which the unanimity rule holds for the representors, Topey's second argument is unsound.

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24) Some may think that in order for a claim like '$\boldsymbol{C}_{H\equiv P}(H) \neq \boldsymbol{C}_{H\equiv \sim P}(H)$' and '$\boldsymbol{C}(P) = \boldsymbol{C}(\sim P)$' to be indeterminate, the denoting terms flanking the (non-)identity sign must be indeterminate in reference, and that the indeterminacy in reference should be explained. It is worth investigating how to understand the indeterminacy in reference in the coin puzzle, and answering the question satisfactorily should lead to interesting discussion of imprecise credences. However, I will leave that for future research.

However, important questions remain that the proponents of imprecise credences need to answer. The response to Topey's first argument seems to assume some substantial roles regarding credence functions. That is, applying ERP to the coin puzzle, each credence function seems to be required to see its own future. Then, how should we understand the epistemological (or metaphysical) status of those credence functions? If credence functions in a representor are merely mathematical fictions to properly represent our ignorance from unspecific evidence, could we still apply the Reflection principle to them? These are intriguing questions that I hope to eventually explore.

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비정밀 신념도 옹호: Topey에 대한 반론

정 재 민

행위자가 명제 P에 대해 충분한 증거를 지니고 있지 않을 때 그 행위자는 P에 대해 어떠한 신념도를 지녀야 할까? Joyce(2010)는 합리적 행위자는 P에 대해 비정밀 신념도를 지녀야 한다고 주장한다. 반면 최근 Topey(2012)는 그러한 Joyce의 관점에 대한 두 가지 흥미로운 반대 논증들을 제시한다. 그러나 본 논문에서 필자는 Topey의 각각의 논증에 허점이 있으며, 따라서 비정밀 신념도를 옹호하는 그 누구도 Topey의 논증들에 의해 설득될 필요가 없음을 보인다.

주요어: 비정밀 신념도, 로저 화이트의 동전 퍼즐, 반영 원리, 만장일치 원리