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Resolving Humphreys' Paradox*

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According to the propensity interpretation of quantum probabilities, initiated by W. Heisenberg and systematically pursued by people like K. Popper and R. Giere, quantum probabilities represent propensities. Against this view, Paul Humphreys(1985) claimed that propensities cannot be probabilities. In this paper, I take up this challenge and argue that Humphreys' argument isn't well-founded. In doing so, first of all, I investigate algebraic structures of the totality of dispositional properties that a quantum system might have, and show that at least 3 distinct algebraic structures, i.e., a family of completely disconnected Boolean lattices, a partial Boolean algebra or a non-Boolean lattice can be associated with this totality of quantum dispositions. Then I investigate how these algebraic structures serve as algebraic bases for defining quantum probabilities. Finally, I discuss Humphreys' argument that propensities cannot be probabilities, and show that his argument, in its current form, fails to hold at least in quantum domain not just because of the alleged failure of distributivity, but more fundamentally because of the existence of incompatible quantum events.

【KEYWORDS】 quantum probabilities, quantum dispositions, propensities, Humphreys' paradox

1. Introduction.

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It is a well-received view that we are given irreducible probabilities in quantum mechanics. We have failed, however, to reach an agreement about what these probabilities represent. One of the competing accounts has been the so-called propensity interpretation of quantum probabilities, according to which these quantum probabilities represent propensities. This was the view, historically speaking, expounded by Werner Heisenberg (1958, 1961), and systematically pursued by Karl Popper (1959, 1982, 1983) and people like Ronald Giere (1973, 1976).[1] However, one serious challenge was brought up against this propensity account of quantum probabilities. That is Paul Humphreys' claim (1985) that propensities cannot be probabilities, which has been dubbed "Humphreys' paradox" by J. H. Fetzer (1981, 283). In this paper, I take up this challenge and argue that Humphreys' argument is not well-founded.

In doing so, first of all, I take the view that a quantum system has certain graded dispositions, i.e., propensities. Then I investigate algebraic structures of the totality of dispositions that a quantum system might have, and the way they serve as algebraic bases for defining quantum probabilities (and in turn, quantum propensities). Finally, I discuss Humphreys' argument that propensities cannot be probabilities and show that his argument, in its current form, fails to hold at least in the quantum domain.[2]

2. Investigating the algebraic structures of quantum event

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[1] For the discussion about a possible connection between a realistic interpretation of quantum mechanics and the propensity interpretation of quantum probabilities, see Cho (1992).

[2] For recent attempts to resolve Humphreys' paradox on the tracks different from mine, see Mccurdy (1996) and Gillies (2000).

space.

The aim of this section is to investigate algebraic structures of the totality of dispositional properties that a quantum system might have. The main motive is that they will serve as algebraic bases for defining quantum probabilities (and hence, propensities). One conventional way of exploring these algebraic structures has been to read off certain algebraic structures from a Hilbert space associated with quantum system. This procedure is the most straightforward way of doing the job, but it presupposes the legitimacy of Hilbert spaces as mathematical models for quantum mechanics. An alternative approach has been to deduce algebraic structures of the set of properties that a quantum system can have (or experimental questions corresponding to these properties) from probing those experimental procedures applicable to the system, and to see whether certain algebraic structures of a Hilbert space associated with the system can be regenerated. The merit of this latter approach is that the legitimacy of Hilbert space models is sought rather than simply presupposed. My way of investigating algebraic structures of quantum event space may be taken as a variant of this latter approach. That is, I begin by taking the view that a quantum system has certain graded dispositions, i.e., propensities. For this is the view I take to underlie the propensity account of quantum probabilities. Then, I ask, what algebraic structures are associated with the totality of these quantum dispositions?

Let us investigate the algebraic structure(s) of dispositional properties attributable to a quantum system. We denote a physical quantity pertaining to a quantum system s and its corresponding self-adjoint operator by the same symbol A. Then a typical probability question in quantum mechanics is this: What is the probability that the value of the physical quantity A lies in E, which is a subset of the real line **R**? But it is impossible to construct a probability measure defined on all subsets of **R**. We should restrict ourselves to a distinguished family of subsets of **R**, i.e., the family of Borel subsets ("*B*(**R**)"), which is the smallest family of subsets of **R** that includes the open sets and is closed under complements and under countable intersections. So, a well-formulated probability question is this: What is the probability that the value of A lies in a Borel set E?[3] Hence, in the rest of this section, we shall focus on the algebraic structure of quantum dispositions which have the form "the disposition of having the value of A in a Borel set E".

First of all, let us consider quantum dispositions related to a single physical quantity A pertaining to the system s, and write the disposition of having the A-value in a Borel set E as 'D[A, E]'. And let '$C_{D[A, E]}$' refer to a proper test condition for D[A, E], and '$M_{D[A, E]}$' the positive test result for D[A, E]. Then we can define D[A, E] as follows:

> Given that $C_{D[A, E]}(s)$ is realizable, D[A, E](s) iff $C_{D[A, E]}(s)$ □→ $M_{D[A, E]}(s)$.[4]

For the quantum system s, $C_{D[A, E]}$ is a proper measurement for the quantity A. Let us take A to be an observable. Then, by definition, an A-measurement is in principle possible, i.e., $C_{D[A, E]}(s)$ is

[3] Cf. Beltrametti & Cassinelli (1981), p. 4.

[4] '□→' is the symbol for subjunctive conditional. In this paper, I'll assume that we have an intuitive understanding of the truth-conditions for subjunctive conditional in use.

realizable. Hence,

(T) "D[A, E](s)" is true iff "$C_{D[A, E]}(s)$ □→ $M_{D[A, E]}(s)$" is true iff the vector |ψ>, which represents the state of s, is a superposition of the A-eigenstates whose corresponding A-eigenvalues belong to E.

Now let us consider a set of quantum dispositional properties related to A, D_A = {D[A, E]: E ⊆ **R** is a Borel set}. The test condition for each disposition belonging to D_A is one and the same, i.e., a proper A-measurement. So all pairs of dispositions belonging to D_A are compatible in the sense that their test conditions are simultaneously realizable. Then, on the basis of the equivalence relation ~, we have a set of equivalence classes, D_A/~. And we can show that D_A/~ (and hence, D_A) has the algebraic structure of a Boolean lattice. Next, we consider a set of quantum dispositions related to a pair of commuting observables, A and B. And let D_A = {D[A, E]: E ⊆ R is Borel} and D_B = {D[B, F]: F ⊆ R is Borel} be the sets of quantum dispositions related to the observables A and B respectively. Further let $D_{\{A, B\}}$ be the closure of $D_A \cup D_B$ under the operations of complementation, *, and conjunction, •. Then we can show that $D_{\{A, B\}}$/~ has the structure of a Boolean lattice. Finally for an arbitrary set of pairwise commuting observables, O = {A, B, C, ...}, we can show that the algebraic structure of D_O/~, where D_O is the closure of $D_A \cup D_B \cup D_C \cup$ ... under * and •, is also a Boolean lattice. Further, in general, it is the case that a quantum system is associated with more than one set of commuting observables. Hence, the system is associated with a family of

Boolean lattices of quantum dispositions.[5]

However, not all the quantum dispositions attributable to s are mutually compatible. In fact, some quantum dispositions are *incompatible* in the sense that the conditions for their manifestation cannot be realized simultaneously. For example, D[S_x, {+½ħ}] (i.e., the disposition of having the spin-value +½ħ in the x-direction) and D[S_z, {+½ħ}] (i.e., the disposition of having the spin-value +½ħ in the z-direction) of the system s consisting of a single electron are incompatible quantum dispositions in the sense that their test conditions, i.e., a S_x-measurement and a S_z-measurement, cannot be realized simultaneously. Can we make sense of such a conjunction of two incompatible quantum dispositions? The problem of understanding the conjunction of two incompatible dispositions amounts to the problem of understanding a disposition whose test condition is not realizable. So, we seem to have two options for understanding the conjunction of two incompatible quantum dispositions. One is to say that such a conjunction is not well-defined, and the other is to say that it is well-defined.

Let us consider, first, the option of saying that the conjunction of two incompatible quantum dispositions is not well-defined. Let D_S be the totality of dispositional properties that a quantum system s might have. We shall be given maximal subsets of D_S of mutually compatible dispositional properties. As mentioned earlier, the quotient of each of these subsets has the algebraic structure of Boolean lattice. We still have two distinct sub-options. One sub-option is to say that these Boolean lattices are completely disconnected in the way that no two of them share a common element(s) or ordering relation. Then we have just a collection of

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[5] For more detailed discussions, see Cho (1990), pp. 51-4 & Cho (1992).

completely disconnected Boolean lattices. The other sub-option is to say that they all share two common elements, 0 and I, and nothing else. Then we have a partial Boolean algebra, and the Boolean lattices in the above-mentioned collection become its sublattices.

Next, the other main option is to take the conjunction of two incompatible quantum dispositions to be well-defined. Speaking in terms of propositions corresponding to dispositions, it holds in general for an orthomodular lattice of propositions of a physical system that if the proposition lattice contains a pair of incompatible propositions, then it contains a pair of nontrivial complementary propositions and hence is non-Boolean.[6] Insofar as we take the conjunction of any pair of incompatible quantum dispositions as well-defined, it isn't difficult to show that the totality of quantum dispositional propositions corresponding to quantum dispositions attributable to a physical system has the structure of an orthomodular lattice. Hence the result of adopting this latter option is that the Boolean lattices under consideration together form a non-Boolean lattice, where they share not only 0 and I but also the ordering relation.

Thus, it turns out that we can associate at least 3 distinct algebraic structures, i.e., a family of completely disconnected Boolean algebras, a partial Boolean algebra, or a non-Boolean algebra, with the totality of dispositional properties that a quantum system might have.

3. Defining quantum probabilities.

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[6] Cf. Beltrametti & Cassinelli (1981), p. 163.

The role of probability statements in quantum mechanics is quite central. For example, all the predictions about measurement results in quantum mechanics are made by probability statements. The theoretical reason is that even a complete specification of the state of a quantum system allows for only probabilistic statements about possible measurement results. So the questions about measurement results take the form of "What is the probability that upon a proper X-measurement, the system s would have a X-value x_i (or have a X-value in E)?". Thus, quantum probability statements are statements that assign probability to quantum events, equivalently, to quantum dispositional propositions.[7] Let us consider a formal setting for quantum probabilities. We need an algebraic structure upon which quantum probability measures can be defined. Let L_S be the quotient of the set of all quantum dispositional propositions that can be associated with a system s. In fact, $L_S = D_S/\sim$. We noted earlier that at least 3 distinct algebraic structures could be associated with L_S. So we have to expect that we shall have more than one formal setting for quantum probabilities.

Suppose that we take the algebraic structure associated with L_S to be a collection $\{B_i\}$ of completely disconnected Boolean (σ-)lattices. According to Loomis'(1947) representation theorem, each Boolean (σ-)lattice B in the collection can be represented by a Boolean (σ-)lattice Σ of subsets of a space Ω. Let α be a nonnegative measure on the Boolean (σ-)lattice Σ satisfying α(Ω) =

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[7] Since quantum probability measures are defined on a lattice of equivalent classes (of quantum dispositions or propositions), it is initially to these equivalence classes (or "events") that quantum probabilities are assigned. However, assigning a probability to an equivalent class amounts to assigning the same value to each member of the class.

1. Then for each Boolean lattice B, we have a classical probability space <Ω, Σ, α>. And a perfect analogy obtains between this classical probability space and a classical probability space describing a classical system. However, as soon as we begin to consider more than one Boolean (σ-)lattice in the collection $\{B_i\}$, the analogy with its classical counterpart breaks down. For example, consider two incompatible quantum dispositional propositions, one of which belongs to B_i and the other of which belongs to B_j ($j \neq i$). Given the structure of $\{B_i\}$, the conjunction of these two propositions is not well-defined. So no probability can be assigned to such a conjunction. We should expect the same thing for the disjunction of two incompatible quantum dispositional propositions. Similarly, the conditional probability for two incompatible quantum dispositional propositions Q_i and Q_j (i.e., the probability that given Q_i, Q_j) is not well-defined. Over all, the standard probability calculus applies within and only within each classical probability space <Ω, Σ, α>. As such, the characteristic feature of quantum probability measures defined on the algebraic structure of $\{B_i\}$ is that they are defined relative to each Boolean (σ-) lattice B_i, and hence, making clear their relativization to B_i, they should be denoted by α_{Bi}. On the other hand, there seems to arise one apparent problem with defining quantum probabilities on the algebraic structure of a family of completely disconnected Boolean lattices. Let the state vector $|\psi\rangle$ represent the state of a quantum system s. And let H_s be the Hilbert space associated with the system. Then in computing quantum probabilities, we use the following algorithm: $\alpha^{|\psi\rangle}(Q) = tr(P_M D)$, where P_M is the projection operator on the closed subspace M of H_s corresponding to the quantum dispositional proposition Q and D is the density operator corresponding to $|\psi\rangle$. Let *L* be the set of all quantum dispositional

propositions (associated with s),[8] *M* the set of all closed subspaces of H_s, and *P* the set of all projection operators on H_s. In using the above-mentioned quantum probability algorithm $\alpha^{|\psi>}(Q) = tr(P_M D)$, we cannot assume either the isomorphism between *L* and *M* or that between *L* and *P*. Assume an isomorphism between *L* and *M*. The latter set has the algebraic structure of a non-Boolean lattice. So this amounts to attributing a non-Boolean lattice structure to *L*. Or assume an isomorphism between *L* and *P*. The latter set has the algebraic structure of a partial Boolean algebra. So this amounts to attributing a partial Boolean algebra structure to *L*. Thus the implication of assuming one or the other is at odds with attributing the structure of a family of completely disconnected Boolean lattices to *L*. We may solve this problem by restricting isomorphism between Boolean lattices of quantum dispositional propositions and those of closed subspaces of H_s (or those of projection operators on H_s). This solution, however, amounts to saying that the Hilbert space models for the set of all quantum dispositional propositions, i.e., the set of all closed subspaces and the set of projection operators, both are overstructured.

The second possible algebraic structure associated with a set of quantum dispositional propositions is the structure of a partial Boolean algebra PBA, where all the Boolean lattices in the collection $\{B_i\}$ share just two elements, i.e., 0 and I. For a PBA, we no longer have something like Loomis representation theorem, which would allow us to represent PBA by a (σ-)algebra of subsets of a space Ω. Hence, we need a *generalized measure* notion, which allows us to define probability measures on a non-Boolean algebra,

[8] More precisely, L is the set of all equivalence classes of quantum dispositional propositions or properties) associated with s.

i.e., which neither requires in advance any specific representation of this algebra as a field of sets nor requires it to be distributive. The conditions for defining probability measures on a PBA are still the same as usual: A real-valued function α on PBA is a probability measure on PBA if

(i) $0 \leq \alpha(Q) \leq 1$ for all $Q \in$ PBA, $\alpha(0) = 0$, and $\alpha(I) = 1$;

(ii) α is countably additive, that is, if $Q_1, Q_2, \ldots$ is a sequence of mutually orthogonal elements of PBA, then $\alpha(\vee_i Q_i) = \sum_i \alpha(Q_i)$.

Now, given the partial Boolean algebra structure of a set of quantum dispositional propositions, we are naturally induced to assume an isomorphism between the set of all quantum dispositional propositions and the set of all projection operators on H. And, the fact that the conjunction and disjunction of two incompatible quantum dispositional propositions are not well-defined corresponds to the fact that the product of two noncommuting projection operators is not a projection operator. Furthermore, it goes without saying that no probability is assigned to various compounds of two incompatible propositions. Correspondingly, the function $P \rightarrow tr(PD)$, where D is a density operator and P is a projector, is not well-defined for the product of two noncommuting projectors. Thus, the idea of associating a partial Boolean algebra structure with a set of quantum dispositional propositions is perfectly in harmony with the use of the quantum probability algorithm "$\alpha^{|\psi>}(Q) = tr(P_M D)$" for computing probabilities of quantum dispositional propositions.

Finally, we can take algebraic structure of a set of quantum dispositional propositions to be a non-Boolean lattice NBL. Then probability measures defined on a NBL are the same as those defined on its PBA counterpart. But unlike the conjunction and disjunction of two incompatible quantum dispositional propositions in PBA, these compounds are taken to be well-defined in NBL. So it seems natural to expect that probability measures defined on a NBL assign probabilities to those compounds. Suppose that M_i, M_j, and M_k are the subspaces of H, which correspond to the quantum dispositional propositions Q_i, Q_j, and Q_k respectively, and further that M_k is the intersection of M_i and M_j. Given a state vector $|\psi>$, probabilities for Q_i, Q_j, and Q_k respectively are well-defined. Suppose that Q_i and Q_j are incompatible. Then in spite of the fact that M_k is the intersection of M_i and M_j, $\alpha^{|\psi>}(Q_k)$ does not represent the probability for the conjunction of Q_i and Q_j. For, as I mentioned above, the product of two incompatible projectors P_{Mi} and P_{Mj} is not well-defined. In other words, the algebraic relation among M_i, M_j, and M_k does not hold either among Q_i, Q_j, and Q_k or among P_{Mi}, P_{Mj}, and P_{Mk}.

So far we investigated the algebraic structures, which can be associated with the totality of dispositional properties that a quantum system might have, and considered how they serve for defining quantum probability measures. In fact our investigation showed us that at least 3 distinct algebraic structures could be associated with the totality of quantum dispositional properties which a quantum system might have. Each such algebraic structure can serve as an event space for defining quantum probability measures. Accordingly, we have at least 3 distinct formal settings for quantum probabilities. Hence, until we come up with good reasons for favoring one algebraic structure over others, we shall have to take these algebraic structures to be on an equal footing.

4. Facing Humphreys' Challenge.

Heisenberg, in expounding a propensity view of quantum properties, suggested that "in modern quantum theory this concept [i.e., the possibility or tendency for an event to take place] takes on a new form: it is formulated quantitatively as probability and subjected to mathematically expressible laws of nature"(1961, p. 9). But he did not elaborate on the precise relationship between propensities and probabilities in quantum mechanics. As a matter of fact, it is Karl Popper who adopted and tried to elaborate the so-called propensity interpretation of probability, which he took to be superior to the traditional frequency interpretation of probability. As Popper himself mentions, he was led to the propensity interpretation of probability especially by the problem of interpreting quantum probabilities. It is generally agreed that the predictions by quantum probabilities must be tested by relative frequencies in a large number of repeated experiments. But, according to Popper, probability statements in quantum mechanics are statements about a single quantum system.[9] And while the

[9] Many standard textbooks for the elementary quantum mechanics contain two kinds of statement about quantum probabilities. On the one hand, they contain statements about the probabilities that a single quantum system has such-and-such values for its physical quantities upon measurements. On the other hand, they suggest that these probability statements must be tested by relative frequencies in a large number of concurrent or repeated experiments. One critical way of understanding this situation has been to say that those probability statements about a single quantum system are misstatements and they must be understood as statements about an ensemble. This view is clearly in line with the ensemble interpretation of quantum states, which says that a state vector describes not a single quantum system but an ensemble of quantum systems. On the other hand, if we take the single system interpretation of quantum states, the probability statements about a single quantum system are legitimate as they are. On this latter view, understanding probability statements about a single quantum system is one thing, and testing them

application of probability theory to single-cases has been a trouble spot for the frequency interpretation of probability, it is, Popper believes, "what the propensity interpretation achieves"(1982, p. 79). Of course, the scope of Popper's propensity interpretation of probability is not restricted to quantum mechanics, but quantum probabilities are its main target, as well as (I think) its stronghold. Furthermore, Popper's view is not that every application of probability fits his propensity interpretation, but that for certain central applications (e.g., quantum mechanics) his propensity interpretation is the best one. For example, he says that:

> I do not even wish to say that the propensity interpretation of probability is the best interpretation of the formal probability calculus. I only wish to say that it is the best interpretation known to me for the application of the probability calculus to a certain type of 'repeatable experiment', in physics, more especially, and also, I suppose, in related fields such as experimental biology. Whether the propensity interpretation is applicable to all cases of betting is also really unimportant to my argument. It has been claimed that it is not applicable to the betting on horses in horse racing. Should this turn out to be so, I should simply recommend that a different interpretation be applied to this case. ... However this may be, the formal probability calculus clearly is applicable to a large class of 'games of chance'. But I am not trying to propose universally satisfactory meanings for the words 'probable' and 'probability', or even a universally applicable interpretation of the formal calculus. I am trying to propose an interpretation of the probability calculus which is not ad hoc, and which solves some of the problems of the interpretation of quantum theory (Popper 1982, p. 69).

So, we may take the strategy to assess a propensity interpretation of probability mainly on the basis of its merits in the quantum

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by relative frequencies is another.

domain.[10] But my concern in this paper is with whether we can coherently pursue the idea that quantum probabilities represent propensities. In this connection, I shall take up one major challenge to a propensity interpretation of quantum probability, namely Humphreys'(1985) claim that propensities cannot be probabilities.

His argument runs roughly as follows:

(1) According to the propensity theories of probability, propensities must have the properties prescribed by probability theory, particularly, they must satisfy the axioms (and in turn, the theorems) of the probability calculus.

(2) Propensities fail to satisfy at least some of the laws of the probability calculus, particularly, the inversion theorems such as the multiplication principle and Bayes theorem.

(3) Hence, propensities cannot be probabilities.

There is no doubt that Humphreys' just-mentioned argument constitutes a serious challenge to those propensity theories, which hold that propensities are probabilities and hence must satisfy the axioms (and in turn, the theorems) of the probability calculus.

When Humphreys argues that propensities cannot be probabilities, he seems to have in mind propensities in general. That is, his examples include a radioactive atom s propensity to decay and a neighbor's propensity to shout at his wife on hot summer days (Humphreys 1985, p. 557). But Humphreys himself

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[10] This seems to be the strategy employed by Popper: it is by its success or failure in this field of application [i.e., quantum theory] that the propensity interpretation will have to be judged. (Popper 1983, p. 352)

admits that "there are well-known reasons for doubting the universal application of distributivity to quantum probabilities"(1985, p. 567), while distributivity is one of the assumptions used in his arguments that inversion theorems of the classical probability calculus, such as the multiplication principle and Bayes theorem, are not always applicable to propensities (1985, pp. 559- 563). Then it immediately occurs to us that if Humphreys had taken the non-Boolean structure of quantum events seriously, he should have confined the scope of his argument to non-quantal propensities. For, assuming the non-distributive structure of quantum events, Humphreys' arguments for the premise (2) simply break down. Can we conclude that the propensity interpretation of quantum probabilities is immune from Humphreys' argument that propensities cannot be probabilities? It is too early, I think, to draw such a conclusion. For the issue is more involved than it might appear to be.

It appears at a glance that taking the option of associating a family of totally disconnected Boolean algebras with the quantum event space, we can no longer appeal to the failure of distributivity in trying to refute Humphreys' argument in question. But it is important to notice that regarding the question of when the law of distributivity holds, three algebraic structures under consideration, i.e., a family of totally disconnected Boolean algebras, a PBA, and a NBL, are squarely in accord. Consider a NBL first. In a non-Boolean lattice, the law of distributivity does not hold in general. Instead it holds in each Boolean sublattice. In a PBA, meet and join are well-defined only for compatible elements. So the question of whether the law of distributivity holds does not arise at all for incompatible elements of the PBA. But again, it holds in each Boolean sublattice of the PBA. Similarly, in a family of totally disconnected Boolean algebras, the question of whether the law of distributivity holds does not arise between Boolean algebras. But, needless to say, it holds in each Boolean algebra. In fact, Boolean algebras in the family correspond to Boolean sublattices of the PBA or of the NBL. Hence, if we can refute Humphreys' argument in question by appealing to the failure of distributivity in a NBL associated with the quantum event space, we should be able to do the same thing when associating a family of totally disconnected Boolean algebras with the quantum event space. However, this does not settle the issue completely. The reason is this. The law of distributivity still holds in each Boolean algebra (or Boolean sublattice). Hence Humphreys' argument in question may go through in a quantum event space associated with each Boolean algebra (or sublattice), unless we have some other reasons for refuting it. So we need to take a closer look at Humphreys' argument that propensities cannot be probabilities, particularly, his discussion involving the cases from the quantum domain.

Following Humphreys, let us consider a quantum mechanical phenomenon, i.e., the transmission and reflection of photons from a half-silvered mirror. Humphreys' "detailed argument" that the multiplication principle fails for propensities runs as follows (1985, pp. 559-563)[11, 12]:

(1) Any standard axiomatic system for conditional probability will contain this multiplication principle:

(MP) p(A&B/C) = p(A/B&C)p(B/C) = p(B/A&C)

[11] Here the notation '~' is used for negation, unlike its earlier use for equivalence relation.

[12] In this paper, following Humphreys, I use 'p' for probability and 'pr' for propensity.

$p(A/C) = p(B\&A/C)$.

Further assume the additivity axiom for conditional probabilities:

(Add) If A and B are disjoint, then $p(A \vee B/C) = p(A/C) + p(B/C)$.

Then from (MP) and (Add) together with the law of distributivity, the theorem on total probability for binary events follows:

(TP) $p(A/C)=p(A/B\&C)p(B/C) + p(A/{\sim}B\&C)p({\sim}B/C)$.

(2) Let I_{t2} be the event of a photon impinging upon the mirror at time t_2, and let T_{t3} be the event of a photon being transmitted through the mirror at time t_3 later then t_2. Now consider the single-case conditional propensity $pr_{t1}(./.)$ where t_1 is earlier than t_2, and take assignments of propensity values:

(i) $pr_{t1}(T_{t3}/I_{t2}\&B_{t1}) = m > 0$

(ii) $1 > pr_{t1}(I_{t2}/B_{t1}) = n > 0$

(iii) $pr_{t1}(T_{t3}/{\sim}I_{t2}\&B_{t1}) = 0$

where B_{t1} is a complete set of background conditions which include the fact that a photon was emitted from the source at t_0, which is no later than t_1. We further assume that:

(CI) $pr_{t1}(I_{t2}/T_{t3}\&B_{t1}) = pr_{t1}(I_{t2}/{\sim}T_{t3}\&B_{t1}) = pr_{t1}(I_{t2}/B_{t1})$.

That is, the propensity for a particle to impinge upon the mirror is unaffected by whether the particle is transmitted or not.

(4) From (TP) together with (i), (ii), and (iii), we have $prt1(T_{t3}/B_{t1}) = mn$.

From (CI), we have $pr_{t1}(I_{t2}/T_{t3}\&B_{t1}) = pr_{t1}(I_{t2}/B_{t1}) = n$.

Hence using (MP) we have $pr_{t1}(I_{t2}\&T_{t3}/B_{t1})$ =

$pr_{t1}(I_{t2}/T_{t3}\&B_{t1})pr_{t1}(T_{t3}/B_{t1}) = mn^2$.

But from (MP) directly we have

$pr_{t1}(I_{t2}\&T_{t3}/B_{t1}) = pr_{t1}(T_{t3}\&I_{t2}/B_{t1}) = pr_{t1}(T_{t3}/I_{t2}\&B_{t1})pr_{t1}(I_{t2}/B_{t1}) = mn$.

We thus have $mn^2 = mn$, i.e., $m = 0$, $n = 0$, or $n = 1$, which is inconsistent with (i) or with (ii).

Let s be a quantum system, associated with a Hilbert space H. Further let P(H) be the set of all closed linear subspaces of H. Then P(H) is in fact projection lattice, since a projection operator one-to-one corresponds to the closed subspace that is its range. Let $|\psi>$ be a state vector which represents the state of s, D be a density operator corresponding to $|\psi>$, and P_M be a projection operator which corresponds to the closed space M. Gleason proved that if the dimension of H is not less than 3, then every probability measure on P(H) arises from a density operator in H, according to $\alpha^{|\psi>}(M) = tr(DP_M)$. In other words, every probability measure on P_M(H) must be relativised to the state of s. Then the initial problem with Humphreys' above argument is that the probability (or propensity) measures used in the latter argument are not explicitly relativised to the state of a photon. This defect can be easily fixed as follows: Humphreys' argument in question use conditional probabilities (and propensities); further in (2), he makes clear that the conditioning event B refers to a complete set of background conditions, which is intended to satisfy the condition of "maximal specificity"; on conventional quantum mechanics, it is taken for granted that the state of a quantum system is completely specified by a state vector; hence, we can replace the conditioning event B_t by a state vector $|\psi(t)>$ which represent the state of a photon at time t. An additional change is needed at this point. That is, probability (and propensity) measures must be relativised to $|\psi(t)\rangle$. The result is that $p^{|\psi(t)\rangle}$ (or $pr^{|\psi(t)\rangle}$) is different from a usual conditional probability (or propensity). Suppose that $p^{|\psi(t)\rangle}(e(X))$ is the same as the conditional probability function $p(./e(|\psi(t)\rangle))$, where $e(|\psi(t)\rangle)$ stands for the event that the photon is in the state $|\psi(t)\rangle$ at time t. The latter function is usually defined as $p(.\&e(|\psi(t)\rangle))/p(e(|\psi(t)\rangle))$. Then both $p(.\&e(|\psi(t)\rangle))$ and $p(e(|\psi(t)\rangle))$ fail to be relativised to a quantum state, and hence, they are not genuine quantum probability measures. On the other hand, $p^{|\psi(t)\rangle}(.)$ is an honest quantum probability measure.

Under the suggested changes, Humphreys' (i), (ii), (iii) and (CI) will have to be written as follows:

(i*) $pr^{|\psi(t1)\rangle}(T_{t3}/I_{t2}) = m > 0$
(ii*) $1 > pr^{|\psi(t1)\rangle}(I_{t2}) = n > 0$
(iii*) $pr^{|\psi(t1)\rangle}(T_{t3}/{\sim}I_{t2}) = 0$
(CI*) $pr^{|\psi(t1)\rangle}(I_{t2}/T_{t3}) = pr^{|\psi(t1)\rangle}(I_{t2}/{\sim}T_{t3}) = pr^{|\psi(t1)\rangle}(I_{t2})$.

The major change in (ii*) is that the propensity involved no longer has the form of usual conditional propensity. But $pr^{|\psi(t1)\rangle}(I_{t2})$ proves itself to be well-defined quantum propensity so that (ii*) holds even after the suggested changes. On the other hand, (i*), (iii*), and (CI*) involve propensities which still have the form of usual conditional propensity. (i*) and (iii*) together, like (i) and (iii), suggest that T_{t3} is causally dependent upon I_{t2} (or ${\sim}I_{t2}$), while (CI*), like (CI), suggests that I_{t2} is causally independent of T_{t3} (or ${\sim}T_{t3}$). Thus (i*), (iii*), and (CI*) together express an asymmetry in the causal connection between I_{t2} and T_{t3}. And this causal asymmetry generates Humphreys' argument under consideration. However, we need to take a closer look at those conditional propensities in (i*), (iii*), and (CI*). Conditional probabilities in quantum mechanics are taken either to be not always well-defined or to be defined in a non-classical manner under certain circumstances. Either way, such a nonclassical situation arises from the fact that there exist incompatible quantum events (or quantum dispositional propositions). The problem with (i*), (iii*), and (CI*) is then that I_{t2} and T_{t3} are incompatible quantum events. For I_{t2} involves a position of the photon at time t_2 and T_{t3} involves another position of the same system at time t_3 (later than t_2); and the different positions of a single quantum system at two different times are incompatible.[13] Take the view that conditional probabilities are not well-defined for incompatible quantum events. Then (i*), (iii*), and (CI*) all can no longer be held to be true. Or take the view that for incompatible quantum events, conditional probabilities are defined in a nonclassical manner.[14] Then (CI*) still cannot be held to be true. For it partly assumes the classical definition of conditional probability.[15] Either way, (CI*) fails. And without (CI*) (and hence, (CI)), Humphreys' reasoning in (3) breaks down. Here Humphreys might say that (i*), (iii*) and (CI*) are not about probabilities but about propensities, and hence they hold regardless of whether probabilistic counterparts are well-defined or not. I

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[13] Let X_i and P_i stand for position-operator and momentum-operator in the Heisenberg picture respectively. Then two position operators at equal times commute. That is, $[X_i(t), X_j(t)] = 0$. But the commutator of the X_i's at different times does not vanish. That is, $[X_i(t), X_i(t_0)] = [P_i(t_0)(t-t_0)/m, X_i(t_0)] = -i\hbar(t-t_0)/m$, where $t \neq t_0$ and m is the mass of a given quantum system (cf. Sakurai 1985, p. 86).

[14] Cf. Beltrametti & Cassinelli (1981), pp. 279-280.

[15] It seems clear that (CI*) assumes the classical definition of conditional probability. That is, $pr^{|\psi(t1)>}(I_{t2}/T_{t3}) = pr^{|\psi(t1)>}(I_{t2}\&T_{t3})/pr^{|\psi(t1)>}(T_{t3}) = pr^{|\psi(t1)>}(I_{t2})pr^{|\psi(t1)>}(T_{t3})/pr^{|\psi(t1)>}(T_{t3}) = pr^{|\psi(t1)>}(I_{t2})$.

doubt, however, that we have good reasons for believing that conditional propensities hold between incompatible quantum events. In the quantum world, our classical intuition often goes wrong.

An alternative way of disposing Humphreys' arguments in question is to point out that the law of distributivity holds just within each Boolean algebra belonging to a family of Boolean algebras associated with the quantum event space. As Humphreys himself admits, the law of distributivity is one of the assumptions used in his arguments that inversion theorems of the classical probability theory, such as the multiplication principle and Bayes theorem, are not always applicable to propensities. As we noted above, however, Humphreys "detailed argument" involves two incompatible events, which do not belong to one and the same Boolean algebra. Hence, his use of distributivity is illegitimate.

Similar arguments can be given against Humphreys' claim that Bayes Theorem fails for propensities, which I will not rehearse here. It may still be objected that we have not shown yet that Humphreys' argument fails for non-quantal propensities and, until we do that, we have to admit that classical propensities cannot be probabilities. However, I take it to be beyond the scope of this paper to ask whether classical propensities are genuine propensities or whether Humphreys' argument still applies to them. Nonetheless, it has been shown that Humphreys' argument (applied to quantum propensities) fails not just because of the alleged failure of distributivity, but more fundamentally because of the existence of incompatible quantum events which is an irrefutable feature of quantum mechanics. This means that Humphreys' argument (applied to quantum propensities) is doomed to failure, regardless of whether we associate the structure of a family of totally disconnected Boolean algebras with the quantum event space or some other structure, i.e., a PBA or a NBL, with it.

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험프리즈 역설의 해결

조 인 래

양자 역학의 확률에 대한 개연적 성향 해석에 따르면, 양자 확률은 개연적 성향을 나타낸다. 이러한 견해는 하이젠베르크에 의해 시작되고 포퍼나 기어리 같은 사람들에 의해 체계적으로 추구된 것이다. 이에 대해 개연적 성향은 확률일 수 없다는 반론이 험프리즈(1985)에 의해 제시되었다. 이 논문에서 나는 험프리즈의 논변이 그의 결론을 지지하지 못한다고 주장한다. 이를 위해 나는 양자 물리계에 부여될 수 있는 개연적 성향들의 총체가 적어도 세 가지 다른 대수적 구조, 즉, 완전히 단절된 부울 격자들의 집단, 부분적 부울 대수 또는 비부울적 격자와 연계될 수 있음을 먼저 밝힌다. 뒤이어 나는 이 대수적 구조가 양자 확률들을 정의하는 데 대수적 기초로 사용되는 방식을 탐구한다. 마지막으로 나는, 앞의 결과들을 토대로 하여, 험프리즈의 논변이 분배성의 결여 때문만이 아니라 보다 근본적으로는 양립불가능한 양자 사건들의 존재 때문에 현재의 형태로는 양자 영역에서 성립하지 않는다고 논변한다.