과학철학 16-1(2013) pp. 81-102

Constructibility of logical principles

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I would like to justify that logical rules are neither a verbal convention nor laws of reality, and that they are results of axiomatization by logicians based on logical structures within their contents of consciousness. But these logical structures of adults should be reduced on psychological structures such as the structure of 'INRC group' proper in formal operations of the adolescent, which in turn should be based on the more fundamental logico-mathematical structures such as 'grouping', the structure proper at the stage of concrete operation, in Piaget's terminology. On the other hand, a necessary condition for the groupings is the conservation that means "the invariance of a characteristic despite transformations of the object or collection of objects possessing this characteristic." But we must remind that the object's permanent character is not based on some transcendental principles, but on the organization of the spatial field which is brought about by the coordination of the child's movement, In a word, distinguishing logico-mathematical and infralogical structures, I want to justify the position of constructivism about logical rules. The former are composed of operations performed on individual objects while spatio-temporal relations are not taken into account, the latter concerning the part-whole relationships within an individual object as a whole, taking their spatio-temporal relations into account. Thus, infralogical structure as psychological and prelogical denotes an earlier

접수완료: 2013.5.7/심사완료 및 게재확정: 2013.5.23/수정완성본 접수: 2013.5.25
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level of what will became logical afterwards. In the conclusion, I will critique firstly the logicism by proving that the law of identity is not in principle a priori, but the result of construction of conservation. Secondly, I will critique the various sorts of nominalistic theory of logic by justifying that the logical rules are not merely a verbal convention, but based on the notion of identity which is constructed throughout the assimilation of objects into schemes by the subject′s actions.

【Key Words】Laws of thought, Logical principles, Constructivism, Conventionalism, Fundamentalism, Group of INRC transformation, Group of grouping, Logicism.

Ⅰ. Introduction

Why are propositions of arithmetic, those of geometry and propositional calculation in ordinary language necessary? It is, we think, because they count on certain principles of logic, the law of identity : A is A ; the law of non-contradiction : nothing can be both A and not-A ; the law of excluded middle : everything is either A or not-A. In fact, these appear to be necessarily true and fundamental, because if they were not true, none of the others could be formulated, or even thought of. These are called the ‘Laws of Thought’ by Aristotle, but not in the way that the traditional laws of association are considered laws of thought. The latter is laws of human psychology describing how people actually think. However, the former is presupposition for all consistent thinking. What is then the status of these logical principles? Are they analytic or synthetic, a priori or a posteriori? Here we enter an arena of considerable controversy.

An Empiricist assumes that they are the very paradigm of cases for analytic statements.[1] For example, denying 'Nothing can be both A and not -A' gets us to suppose the statement that A is not-A. Nothing could be more obviously self-contradictory than this statement. Without except, a rationalist claims that the term of 'analytic' can be defined with reference to the Law of Non-contradiction itself. Therefore, this principle is the criterion for the self-contradiction of the other statements. Therefore, rather than saying that the law is analytic, they would prefer to say that it stands outside the system of statements, providing a touchstone whereby they can be tested as analytic. Nevertheless, rationalists do not obviously say that they are synthetic, although they presume that they are a priori. I will try to prove positively that they are both a posteriori and synthetic.

The Empiricist assumes also that the logical principles state only the verbal convention that makes other statements analytic. For example, "cats are cats" is a special case of "A is A," and "This is not both a cat and not a cat" is a special case of "Not both A and not-A." Nevertheless, the rationalist does not agree with the empiricist who says that "Not both A and not-A" is merely a verbal convention, but holds that the so-called principles of thought are fundamental laws of reality. In other words, they do not merely tell us to use words a certain way, but they tell us something about the nature of things. They inform us of certain general facts about reality.[2]

However, I would like to justify that logical rules are neither a verbal convention nor laws of reality, and that they are results of

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1) See John Hospers, 'The principles of logic', *An Introduction to Philosophical Analysis*, Prentice Hall, 1973, p. 217.

2) Ibid., pp. 217-226.

axiomatization by logicians based on logical structures within their contents of consciousness. But these logical structures of adults should be reduced on psychological structures such as the structure of 'INRC group' proper in formal operations of the adolescent, which in turn should be based on the more fundamental logico-mathematical structures such as 'grouping', the structure proper at the stage of concrete operation, in Piaget's terminology. On the other hand, a necessary condition for the groupings is the conservation that means "the invariance of a characteristic despite transformations of the object or collection of objects possessing this characteristic."[3] But we must remind that the object's permanent character is not based on some transcendental principles, but on the organization of the spatial field which is brought about by the coordination of the child's movement, In a word, distinguishing logico-mathematical and infralogical structures, I want to justify the position of constructivism about logical rules. The former are composed of operations performed on individual objects while spatio-temporal relations are not taken into account, the latter concerning the part-whole relationships within an individual object as a whole, taking their spatio-temporal relations into account.[4] Thus, infralogical structure as psychological and prelogical denotes an earlier level of what will became logical afterwards.[5] In the

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3) Piaget, J. 'Quantification, conservation and nativism', Science, 1968, p. 978.

4) See Piaget, J & Inherder, B., *Le développement des quantités chez l'enfant*, Neuchâtel : Delachaux & Niestlé, 1941, p. 332.

5) Some authors as Vuyk, Rita do not think that the infralogical structure of Piaget denote a pre-logical structure. But I think that the structure of grouping is previous to that of INRC, which is previous to the well formed expressions of propositional logics, at least in the psycho-genetical dimension.

conclusion, I will critique firstly the logicism by proving that the law of identity is not in principle a priori, but the result of construction of conservation. Secondly, I will critique the various sorts of nominalistic theory of logic by justifying that the logical rules are not merely a verbal convention, but based on the notion of identity which is constructed throughout the assimilation of objects into schemes by the subject's actions.

Ⅱ. Proposition and classes

The two most important sorts of logical calculation in human being are logics of proposition and those of classes. In fact, human being must have talked each other and calculated his domain where he does live or occupy to survive against wild environment from beginning of social life. Before structuralistic interpretation of logic, most logicians thought that the logics of proposition were deferent from those of classes. Nowadays it has been evident that two logics have been based on the same principles. Therefore, it might be convenient to show this fact just before proving that we can reduce logico-mathematical structures to psychological structures. For if there are various kinds of logics irreducible among them, it would not be perfect for me to try to reduce the logical structures to the psychological ones, even if I had succeeded in getting the laws of propositional inferences based on the psychological structures proper in concrete operations of actions.

We can directly see the isomorphism between the calculation of classes and that of propositions. Therefore, in the following table, the left expressions are the theorems of the calculation of the classes, while the right expressions are of the tautologies of propositional calculus :

1. $\alpha \cup \beta = \beta \cup \alpha$ $\vdash p \vee q \equiv q \vee p$ commutativity of the disjunction
2. $\alpha \cap \beta = \beta \cap \alpha$ $\vdash p \wedge q \equiv q \wedge p$ commutativity of the conjunction
3. $\alpha \cup \beta = \alpha$ $\vdash p \vee p \equiv p$ idempotent of the disjunction
4. $\alpha \cap \beta = \alpha$ $\vdash p \wedge q \equiv p$ idempotent of the conjunction
5. $--\alpha = \alpha$ $\sim\sim p \equiv p$ principle of the double negation

On the other hand, we can see evidently the isomorphism between the calculation of classes and that of propositions throughout a formal deductive system of Boolean algebra that can be set forth as follows[6) :

Special undefined primitive symbols : C, $\cup$, $\cap$, $-$, α, β, γ,…
Axioms :
Ax. 1. If α and β are in C, then $\alpha \cup \beta$ is in C.
Ax. 2. If α and β are in C, then $\alpha \cap \beta$ is in C.
Ax. 3. There is an entity 0 in C such that $\alpha \cup 0 = \alpha$ for any α in C.
Ax. 4. There is an entity 1 in C such that $\alpha \cap 1 = \alpha$ for any α in C.
Ax. 5. If α and β are in C, then $\alpha \cup \beta = \beta \cup \alpha$.
Ax. 6. If α and β are in C, then $\alpha \cap \beta = \beta \cap \alpha$.
Ax. 7. If α, β, γ are in C, then $\alpha \cup (\beta \cap \gamma) = (\alpha \cup \beta) \cap (\alpha \cup \gamma)$.
Ax. 8. If α, β, γ are in C, then $\alpha \cap (\beta \cup \gamma) = (\alpha \cap \beta) \cup (\alpha \cap \gamma)$.
Ax. 9. If there are unique entities 0 and 1 satisfying Axioms3 and 4, then for every α in C there is an $-\alpha$in C such that $\alpha \cup -\alpha = 1$ and $\alpha \cap -\alpha = 0$.
Ax. 10. There is an α in C and a β in C such that $\alpha \neq -\beta$.

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6) See Irving M. Copi, *Symbolic Logic*, Macmillan Publishing Co., Inc., New York, 1979, p. 175.

'C' is the collection of all classes, '0' and '1' are the empty and universal classes, respectively and the symbols '∪', '∩' and '−' represent class addition, multiplication, and complementation, respectively. We will find it so easy to derive all the theorems of logics of classes from these axioms. Therefore, we can say evidently that the calculation of classes is based on the structure of a Boolean algebra. On the other hand, We will find it also so easy to derive some theorems of propositional logics from these axioms. We can interpret 'C' as the collection of all propositions, and 'α', '∩', 'ɣ', … as symbolizing proposition, and interpret '∪', '∩', and '−' as symbolizing conjunction, disjunction, and negation. Then if we further interpret the equals sign(=) as symbolizing material equivalence(≡), all axioms and theorems of the Boolean Algebra become logically true propositions of the propositional calculus. Hence we can say that the propositional logics are based on the structure of Boolean Algebra.

	logics of classes	logics of proposition
$\alpha \leq \beta$	$\alpha \cap \beta = \alpha$, or : $\alpha \subseteq \beta$	the tautology '$p \supset q$', or $\vdash p \supset q$
o	the empty class, or : Ø	an always false proposition, or : $p \wedge \sim p$.
I	the universe of discussion, or : U	an always true proposition, or : $p \vee \sim p$.

On the other hand, we can demonstrate that the logics of classes and of propositions are both a lattice of Boole, just as both two logics are based on the structure of Boolean algebra. The lattice(réseau, treillis) is a structure composed of 4 elements : <E, ∪, ∩, ≤>, which has the following properties :

(L′1) $\alpha \leq \alpha \cup \beta$ (L″1) $\alpha \cap \beta \leq \alpha$

(L′2) $\beta \leq \alpha \cup \beta$ (L″2) $\alpha \cap \beta \leq \beta$

(L′3) $\alpha \leq \gamma$ and $\beta \leq \gamma . \Rightarrow . \alpha \cup \beta \leq \gamma$ (L″3) $\gamma \leq \alpha$ and $\gamma \leq \beta . \Rightarrow . \gamma \leq \alpha \cap \beta$.

(L′4) $\alpha \cap (\beta \cup \gamma) \leq (\alpha \cap \beta) \cup (\alpha \cap \gamma)$ (L″4) $(\alpha \cup \beta) \cap (\alpha \cup \gamma) \leq \alpha \cup (\beta \cap \gamma)$

(L′5) There is an element 1 in E such that for every α, '$\alpha \leq 1$'.

(L″5) There is an element 0 in E such that for every α, '$0 \leq \alpha$'.

(L′6) There is an element ā in E such that for every α, '$I \leq \alpha \cup \bar{a}$'.

(L″6) There is an element ā in E such that for every α, '$\alpha \cap \bar{a} \leq o$'.

Here we can interpret the symbols '∪', '∩', and '−' as symbolizing the union and the disjunction, the intersection and the conjunction, the complementation and the negation, respectively. However, it is not easy to interpret the relation '≤' and the symbols '0' and '1'. Nevertheless, we can proceed like this[7] :

We can see here that the presence of an order between the elements, though classes or propositions, is what is essentially new. This property is particularly manifest for the la logics of propositions. The operations '∨', and '∧' permit to derive the new other propositions from the given. Because of '$\vdash p \supset (p \vee q)$', we can say that 'p' implicate '$p \vee q$', but not that '$p \vee q$' implicate 'p'. The relation that unites the propositions is asymmetric. The method to analyze the logics of propositions in this way allows knowing about what is the motor of reasoning. Nevertheless, this analysis do not let us see what is the source of coherence. That is why we need to analyze the structure of group.

The notion of group is probably one of the most important rules of algebra, because it permits us to manifest more clearly the nature of logics in classes and of propositions. A group can be

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[7] See Jean-Blaise Grize, 'Historique. Logique des Classes et Des Propositions. Logique des Prédicats. Logique Modales' in *Logique et Connaissance Scientifique*, Gallimard, 1967, p. 275.

defined as a triple : <E, o, =>. That has the properties like this[8] :

(G1) x o (y o z) = (x o y) o z
(G2) There is an e∈ E such that e o x = x = x o e.
(G3) There is a x′ such that x′ o x = e = x o x′.
(G4) x o y = y o x.

Therefore, the main characteristic natures of group are association, commutation, the existence of a neutral element and of an inverse element. What is here the most important is the existence of an inverse element, because it can guarantee the possibility of a return to the point of departure, which means a kind of internal coherence. It is evident that both the calculation of proposition and of classes have association and commutation. However, it is easy to verify the existence of the neutral element and of an inverse element in logics of classes, but not in that of propositions. We can proceed like this[9] :

	first possibility	second possibility
operation o	W	≡
inverse element x'	p'.=df. p	p'.=df. p
neutral element e	e.=df. p w p	e.=df. $p \equiv p$
relation $x = y$	$\vdash p \equiv q$	$\vdash p \equiv q$

It is easy to verify that ′ p w p′ is 0, while ′$p \equiv q$′ is 1. The symbols ′0′ and ′1′ are neutral elements(e). We also can verify easily the correspondent($\vdash p \equiv q$)of equal relation(x=y), as we see in the table. But We must not confuse the operation ′≡′ which

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8) Ibid., p. 275.
9) Ibid., p. 277.

constructs the proposition '$p \equiv q$' from the propositions 'p' and 'q' with the relation '$\vdash p \equiv q$' which affirms that a certain proposition '$p \equiv q$' is a theorem. When it approaches the inverse element, it is not easy to find its correspondent in the logics of propositions, as long as it keeps the properties of structure of group, 'x' o x = e = x o x''. So, we must suppose 'p w p' = 0', and 'p ≡ p' = 1'. Therefore, 'p' . = df. p'. In other words, the propositional interpretation of group has led us to identifying 'p' with '~p'. Therefore, the analysis that we have done is not complete : the negation '~p', which was explicitly in the lattice of Boole, has here disappeared. However, in fact, it is hidden rather than inexistent. It is the notion of commutative ring that makes us to see such an element as possessing at the same time the positive and the negative.

The ring is a kind of quaternary, <E, o, *, => that has the following properties :

R 1. <E, o, => is a commutative group
R 2. x*(y*z) = (x*y)*z
R 3. x*(yoz) = (x*y)o(x*z)
R 4. x*y = y*x

If we apply the same conventions to the two examples of group, <E, W, ∧, => and <E, ≡, ∨, =>, these are the commutative rings. Here, we can prove the theorems such that '⊢ 1 ∧ p ≡ p' and '⊢ 0 ∨ p ≡ p'. For '1 ∧ p' is not other than 'p' by the definition itself of group, 'e o x = x = x o e', and just likely '0 ∨ p' is 'p'. On the other hand, as we can see in 'x' o x = e = x o x', there is the negation in the definition of the symbols of '0' and '1'.

Ⅲ. The logical structure and the infralogical structure

To justify the position of constructivism about logical rules is, in the end, to prove the existence of infralogical structure as psychological and prelogical level. While the logico-mathematical structures are composed of operations performed on individual objects without considering their concrete spatio-temporal relations, the psychological structures concern the part-whole relationships within an individual object as a whole, with taking into account their real spatio-temporal relations. In other words, we have to discern empirical abstractions that lead to inductive generalizations from reflective abstractions which lead to constructive generalizations. Empirical abstraction extracts information from the objects themselves, retaining some of their properties while excluding others. Thus inductive generalization is purely extensional, leading from "some" to "all".[10] But reflective abstraction is not directly related to objects but to actions and operations of the subject. It always consists of two components : projective reflection and reconstructive reflection. The former consists of a projection on th a higher plane B of what was taken from the lower plane A, while the latter reconstructs at B what was already present at A. The essential point is that the reconstruction leads to a new structure, that is to say, a qualitative change is made.[11]

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10) See Vuyk, Rita, *Piaget's Genetic Epistemology*, Vol 1, p.119

11) See Piaget, J., Recherches sur l'abstraction réfléchissante. Vol. I : L'abstraction des relations logico-arithmétique. Vol. II : L'abstractions de l'ordre des relations spatiales. *E. E. G.*, Vols XXXIV and XXXV. Paris : P. U. F., 1977, p. 6.

1. The stage of formal operation and the structure of INRC group

We can see the best example of such a psychological structure through a group of four transformation INRC on that Jean Piaget had thrown light. According to Piaget, INRC is a formal structure that is observed in the material action of that adolescent at the stage of formal operation in the terminology of Piaget. INRC is the structure of transformation which we can operate with 4 elements , 'a', 'b', 'c', 'd', each of which denotes the value '1' or the value '0'. Elements 'a'', 'b'', 'c''and 'd'' designate the opposite value to that of 'a', 'b', 'c', 'd'.[12]

1) Identical transformation I(abcd) = abcd ; I(1011) = 1011.
2) Inverse transformation N(abcd) = a'b'c'd' ; N(1011) = 0100.
3) Reciprocal transformation R(abcd) = dcba ; R(1011) =1101.
4) Correlative transformation C(abcd) = d'c'b'a' ; C(1011) = 0010

We can see now that the composition of any two of the operations gives the third one and the combination of all three again gives the identical I : NR(abcd) = N(dcba) = d'c'b'a' = C(abcd) ; N(RC) = NN = I ; (NR)C = CC = I. Therefore, the INRC group is a specific case of the general Klein group.

	I N R C
I	*I N R C*
N	*N I C R*
R	*R C I N*
C	*C R N I*

Thus we can prove that this INRC group satisfies the axioms

12) See Jean-Blaise Grize, op. cit., p. 284.

that define a commutative group.[13)]

G1 : X(YZ) = (XY)Z ; It is evident on this : N(RC) = NN = I ; (NR)C = CC = I.

G2 : There is an element I such that IX.= X. ; It is evident in the table.

G3 : There is an element X′ such that for every X, X′X = I ; It is evident on this : CC = I ; NN = I ; RR = I ; II = I. So, here X = X′.

G4 : XY = YX. ; Thus, we can see evidently this property, concerning the diagonal of the table.

Now, we can show how the INRC group can generate 16 propositions of normal disjunctive forms. Given two propositions ′p′ and ′q′, each of which can be either true or false, then we can have four propositions as following, that we will call elementary : $p1 =$ df. $p \wedge q$; $p2 = p \wedge \sim q$; $p3 = \sim p \wedge q$; $p4 = \sim p \wedge \sim q$. The sign ′=df.′ is an abbreviation for ′equal by definition′. Let us now make all the possible disjunctives with these elementary propositions like the following table, and calculate their evaluation. Then, we will obtain the table below[14)] :

4 to 4	1	p1∨p2∨p3∨p4	1111	0000		16	0 to 0
3 to 3	2	p1∨p2∨p3	1110	0001	p4	15	1 to 1
	3	p1∨p2 ∨p4	1101	0010	p3	14	
	4	p1∨ p3∨p4	1011	0100	p2	13	
	5	p2∨p3∨p4	0111	1000	p1	12	
2 to 2	6	p1∨p2	1100	0011	p3∨p4	11	2 to 2
	7	p1 ∨p4	1001	0110	p2 ∨p3	10	
	8	p1 ∨p3	1010	0101	p2 ∨p4	9	

13) Ibid., p. 285.
14) Ibid., p. 187.

Alternatively, we can confirm the remarkable resemblance between the expressions of normal disjunctive forms and those of their evaluations. To all elementary propositions which figure in the table correspond an ′1′ in his evaluation and to all elementary propositions which do not figure in the table correspond a ′0′ in the evaluation. Therefore, we can see that the normal form and the evaluation represent a same reality. Let me consider, for example, the normal form 4 and write it explicitly. We have : $(p\wedge q)\vee(\sim p\wedge q)\vee(\sim p\wedge \sim q)$. The valuation of '$p\supset q$' is '1011'. So, we can obtain the following tautology : $\vdash (p\wedge q)\vee(\sim p\wedge q)\vee(\sim p\wedge \sim q) \equiv (p\supset q)$. Through this way, we can finally have 16 propositions.

On the other hand, we can prove that these 16 propositions can be generated by the INRC group that Piaget had defined :

1) The valuation of '$p\supset q$' is '1011', and I(1011) = 1011. So, $I(p\supset q) = p\supset q$.
2) N(1011) = 0100, which is the valuation of '$p\wedge \sim q$'. Nevertheless, '$p\wedge \sim q$' is equivalent with '$\sim(p\supset q)$'. So, $N(p\supset q) = \sim(p\supset q)$.
3) R(1011) = 1101, which is the valuation of '$q\supset p$'. So, $R(p\supset q) = q\supset p$.
4) C(1011) = 0010, which is the valuation of '$\sim p\wedge q$'. However, $\vdash(\sim p\wedge q) \equiv \sim(q\supset p)$. So, $C(p\supset q) = \sim(q\supset p)$
5) I(1111) = 1111, which is the valuation of '$(p\wedge q)\vee(p\wedge \sim q)\vee(\sim p\wedge q)\vee q(\sim p\wedge \sim q)$'. Thus, N(1111) = 0000, which will implicate all contradictory propositions.

Through this way, we have the below table[15] :

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15) Ibid., p. 286.

I	N	R		C	
1	0	1	0	0	1
$p \vee q$	$\sim p \wedge \sim q$	$p \mid q$	$p \wedge q$	$p \wedge q$	$p \mid q$
$q \supset p$	$\sim p \wedge q$	$p \supset q$	$p \wedge \sim q$	$p \wedge \sim q$	$p \supset q$
$p \supset q$	$p \wedge \sim q$	$q \supset p$	$\sim p \wedge q$	$\sim p \wedge q$	$q \supset p$
$p \mid q$	$p \wedge q$	$p \vee q$	$\sim p \wedge \sim q$	$\sim p \wedge \sim q$	$p \vee q$
p	$\sim p$	$\sim p$	p	p	$\sim p$
$p \equiv q$	$p \text{ w } q$	$p \equiv q$	$p \text{ w } q$	$p \text{ w } q$	$p \equiv q$
q	$\sim q$	$\sim q$	q	q	$\sim q$
N	I		R		C

On the other hand, Gottschalk(1953) and Apostel(1963) proved that the structure of INRC group can be defined on the more general algebra of Boole. If the sign '=' means 'having the equivalent evaluation', we can define the four transformations this way[16] :

$I(p) = p$ $\quad$ $N(p) = \sim p$

$I(\sim p) = \sim I(p)$ $\quad$ $N(\sim p) = \sim N(p)$

$I(p \vee q) = I(p) \vee I(q)$ $\quad$ $N(p \vee q) = N(p) \wedge N(q)$

$I(p \wedge q) = I(p) \wedge I(q)$ $\quad$ $N(p \wedge q) = N(p) \vee N(q)$

$R(p) = \sim p$ $\quad$ $C(p) = p$

$R(\sim p) = \sim R(p)$ $\quad$ $C(\sim p) = \sim C(p)$

$R(p \vee q) = R(p) \vee R(q)$ $\quad$ $C(p \vee q) = C(p) \wedge C(q)$

$R(p \wedge q) = R(p) \wedge R(q)$ $\quad$ $C(p \wedge q) = C(p) \vee C(q)$

These definitions lead to the same results that those of the precedent paragraph did. For example, using the laws of calculation of propositions such as '$p \supset q. = \text{df. } p \vee q$' and '$p \equiv q. = \text{df.} (p \supset q) \wedge (q$

16) Ibid., p. 286.

$\supset$p)', we can have : R (p$\supset$q) = R ($\sim$p$\vee$q) = R ($\sim$p) $\vee$ R (q) = $\sim$R (p) $\vee$ R (q) = $\sim\sim$p$\vee\sim$q = p $\vee$ $\sim$q = $\sim$q$\vee$p = q$\supset$p.

2. The stage of concrete operation and the structure of grouping

According to Piaget, while the main structure of the logics of classes and of the logics of propositions of the adult is that of the algebra of Boole, and the psychological structure which is characteristic at the stage of formal operation of the adolescent is that of INRC group, it is the structures of grouping that is characteristic at the stage of concrete operation. Alternatively, the latter is the most important structure, because it is the first one that is basic for formal operations and thereby for scientific operations of the n^{th} degree.

The structure of grouping is a quaternary $\langle E, +, -, \leq \rangle$, where E is un ensemble of objects partially ordered by the relation $\leq$(by definition, the relation $\leq$ is therefore transitive, reflexive and antisymmetric) and where '+' and '−' designate two binal operations. Far more, we have the following properties[17] :

1) The operation '+' is defined only between certain elements of totality. It is absolutely necessary that 'x' be immediately less than 'y' by the relation '$\leq$'(or the inverse), or that there be certain chains of elements 'x^1', . . ., 'x^n', such that 'x' be immediately less than 'x^1', 'x^1' less that 'x^2', etc., 'x^n' less than 'y'(or the inverse), in oder that, if x, y $\in$ E, than x + y $\in$ E. In other words, the operation '+' can be executed from approach to approach. This restriction is characteristic at the stage of concrete operation.
2) The operation '+' is associative.

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17) Ibid., p. 281.

3) The operation '−' is the inverse of the operation '+'. It is therefore submitted to the analogous limitations. However, we must keep in mind that the conditions 2 and 3 assure the coherence of the system at least in the psychological point of view, because these justify at the same time both the detour and the return at the point of departure.
4) There is the neutral element, '0', such that for every $x \in E$, $x + 0 = x = 0 + x$.
5) The operation '+' is idempotent.
6) The operation '+' is the one such that if $x \leq y$, than $x + y = y$.

We can derive the two logical significance from these analyses. First, We can demonstrate that the structure of grouping contains a semi-lattice and some properties of group. It is evident that the double aspect of serial and of equational, fundamental at the logics of adults, is already in the germ at the stage of concrete operation. Secondly, according to the genetic epistemology of Piaget, the operation of child is the one of classes or the one of serialization. Nevertheless, the child can neither classify without serialize, and nor serialize without classify. By this characteristics is explained the property of duality which are present both at the logics of classes and the one of propositions. Thirdly, we can therefore say that the structure of grouping is enlarged and synthesized into a Boolean algebra, through the mediation of the INRC structure.

Ⅳ. Conclusion

So far, First, I tried to prove that the propositional calculus is a Boolean algebra, just as the calculation of class inclusion is so.

Therefore, I tried to demonstrate the isomorphism in the structural point of view between the logic of classes and the one of propositions. Alternatively, you may object that there is not in the logics of classes what is correspondent to the operation '≡' in the logics of propositions. For the two signs '≡' which is appears in the tautology such as $\vdash (p \equiv q) \equiv [(p \wedge q) \vee (\sim p \wedge \sim q)]$, has a very different interpretation. While the second expresses a relation, the tautological equivalence between the proposition of right member and the one of left member, the first gives a new proposition, '$(p \wedge q) \vee (\sim p \wedge \sim q)$'. In other words, '$(p \wedge q) \vee (\sim p \wedge \sim q)$' is a first proposition, and '$\vdash (p \equiv q) \equiv [(p \wedge q) \vee (\sim p \wedge \sim q)]$' is a second. The sign '≡' which can appear as an operator having never any correspondent in the calculation of classes, can be completely eliminated by the only operators '∧', '∨' and '~'. Finally, '$p \equiv q$' is only an abbreviation for the normal form '$(p \wedge q) \vee (\sim p \wedge \sim q)$'. Therefore, that the logic of propositions is richer than the one of classes appears illusionary.

Secondly, but these systems such as a Boolean Algebra, propositional calculus and classes are logico-mathematical structures which are composed of operations performed by logicians or mathematicians on individual signs of symbols while spatio-temporal relation are not taken into account. Alternatively, I have tried to show that these structures are in turn based on psychological infralogical structures thath concern the part-whole relationships within an individual object as a whole, taking their spatio-temporal relations into account. In other words, the logics of propositions appear to be as the synthesis of the two fundamental structures(the grouping of classes and the one of serialization) which precede them genetically. This implies that the operation of classes and the one of seriation can be perceived by empirical abstraction. Therefore, the perception is not a hindrance, but service for conservation. However, there are limits to the use of signifying implication in the task of class-inclusion or of serialization. Therefore, here the implication is qualitative, because *any* quantity has to be constructed. The important aspect is that primitive systems of significations are seriously limited because of the lack of negations, especially negations by constructed by subject and not imposed externally, and that conceptualization could develop together with, but never be more advanced than actions.

Thirdly, I have tried to justify the constructivism. Nevertheless, I deny the one such as logical positivism or analytic philosophy that Popper, Quine and Apostel are assuming. Because they assume the constructivism in the direction of a very hesitating or implicit. In other words, they try to always grasp not the active character, but the static of the logico-mathematical rules and categories, which are susceptible to contact with an apriorism. As a constructivist, I raise the question of whether the subject is the only one responsible for what is constructed or whether the subject is influenced by reality. I suppose that there must be an organism(subject) and an environment(object) with an interaction between the two. Nevertheless, I do suppose neither the realism of empiricists nor Kant's "Ding an Sich" or "noumenon." I suppose that the subject and the reality is in the dialectical relation, in that, due to his cognitive progress, he gets to know the object better and better, but every step of this progress leads to new problems because the object becomes more and more complex. So far, I have tried to combine constructivism with a type of realism in which the subject and the object constantly interact. In a word, while the scheme for construction is the element a priori of logical rules, reflexive abstraction gives us not only knowledge, but also contradiction about the physical world. This contradiction makes us to rectify the previous mode of reasoning and logical rules. That is why I say about the element a posteriori of logical rules, in that the object influences the subject.

Fourthly, I suppose that all the distinctions of logical calculation are based on the negation. It is evident that each of the transformations 'N', 'R' and 'C' can be considered as a sort of negation or complement. By induction, in fact, on the construction of a well-formed expression, we can see that its reciprocal is obtained by the negation of each of its atomic propositions, and that its correlative is not other than its dual, and that its inverse is the reciprocal of its dual. Finally, each of all the constructions is a instantiation of the principle : "omnis determinatio is negatio." The principle of identity which is fundamental and essential in logical thoughts is not given to us form the beginning, but the result of construction for a long time. The object's permanent character(conservation), which is a designation of identity, results from the organization of the spatial field that is brought about by the coordination of the children's movement. The principle of identity is thus based on the operational reversibility, which is in turn based on empirical reversibility. The essential difference between the two is that empirical reversibility is no more than a return to the starting point, centered on the result of the action and without implying the identity of the paths followed. In contrast, operational reversibility is a return that may take place in thought, in which the paths are identical, their opposite directions being the only difference. My conclusion is that the element a posteriori of logical rules are previous to its element a priori, and that logical rules are not fixed, but constructed progressively and gradually with experience of internal or external contradictions.

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