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Instantaneous velocity and the causal explanation problem[†]

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Velocity reductionism is the view that the instantaneous velocity of a body at an instant t is reducible to a relation among the body's positions at the various moments in a neighborhood of t. On the other hand, *velocity primitivism* takes a body's instantaneous velocity at t as an intrinsic property belonging at that instant to the body, which is as primitive as the body's position at t.

Regarding the debate between these two views, this paper aims to do three things: First, I present several examples illustrating the philosophical and physical significance of this debate. Second, I argue that, contrary to common beliefs of many physicists and philosophers, the limit of average velocities is not truly instantaneous. And third, Marc Lange (2005, 2009) recently claimed that instantaneous velocity, if velocity reductionism is correct, cannot play the causal and explanatory roles that classical physics is often interpreted as demanding of them. I

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argue that Lange's criticisms on velocity reductionism are unfounded, i.e., that the causal and explanatory roles can be ascribed to instantaneous velocity in accordance with velocity reductionism.

【Key Words】 velocity, instantaneous velocity, the causal explanation problem, limit, limit property, infinity

1. Introduction

One of the oldest problems in philosophy is about change. The ancient Greek philosopher Heraclitus claimed that change is the only reality and the appearance of persisting things is a mere illusion. Observing the 'same' world, Parmenides insisted on the opposite: The change and diversity of things are illusions and there is only the one being that never changes. Following Parmenides and noting the importance of motion as the prototypical example of change, Zeno of Elea devised his famous paradoxes in an attempt to prove the impossibility of motion and, ultimately, change.

A proper understanding of change and motion calls for deep reflections on the most fundamental fabric of the universe and their properties: space, time, continuity, and infinity. Unlike many philosophers and physicists, I believe that various puzzles of infinity including Zeno's paradoxes have not been fully resolved and still raise fundamental conceptual problems, a proper resolution of which may induce (and require) a new revolution in our understanding of the universe. Yet the discussions of this short essay focus on a small piece of this big puzzle, namely, the nature of instantaneous velocity.

According to one venerable tradition, a change is merely having different properties at different instants, and instantaneous velocity is defined to be, reducible to, and nothing over and above, the limit of average velocities. And the limit of average velocities is not an intrinsic property of a body at a given instant. It is a relation among the body's positions at the various moments in a neighborhood of the given instant or a property of the body's trajectory on an interval surrounding that instant—henceforth, these two ways of defining instantaneous velocity (that is, as a relation and as a property) will be used interchangeably. I will call this view *velocity reductionism*, in short, *reductionism*.

On the other hand, many philosophers (e.g., Tooley 1988, Bigelow and Pargetter 2000, Carroll 2002) have challenged velocity reductionism and advocated an alternative, which is to be called *velocity primitivism*, in short, *primitivism*. According to this view, a change is having a property of change at a given instant t, to be in motion at t is to have a non-zero instantaneous velocity at t, and a body's instantaneous velocity at t is an intrinsic property belonging at that instant to the body, which is as primitive as the body's position at t. Thus, a body's instantaneous velocity at t is irreducible to a relation among the body's positions at the various moments in a neighborhood of t. In those possible worlds where the laws of classical mechanics are natural laws, the intrinsic instantaneous velocity of any body at any instant coincides with the limit of average velocities of the body at that instant as long as the body is moving on a smooth trajectory, but this coincidence is merely due to the physical laws, not a logical connection. And the intrinsic instantaneous velocity at t explains why the object takes up certain positions after t.

Recently, Marc Lange (2005, 2009) criticized both of these two views and proposed an alternative. He claimed that instantaneous velocity and instantaneous acceleration, if velocity reductionism is correct, cannot play the causal and explanatory roles that classical physics is often interpreted as demanding of them. On the other hand, he argued, velocity primitivism fails to recognize that instantaneous velocity is not merely nomologically connected to trajectory, but rather is essentially something to do with trajectory. To capture velocity's essentially kinematic character along with velocity's causal and explanatory roles, Lange made a radical proposal: that instantaneous velocity is roughly akin to a dispositional property.

In this essay I aim to do three things. First, I argue that Lange's criticisms on velocity reductionism are unfounded, that is, that the causal and explanatory roles can be ascribed to instantaneous velocity in accordance with velocity reductionism (Section 4). Yet my ultimate aim does not lie in defending velocity reductionism but rather in making clear various conceptual issues regarding velocity and illustrating the significance of this debate in physics and philosophy. Secondly, thus, I clear up one common misconception about velocity, namely, the conception that the limit of average velocities is truly instantaneous. I refute the arguments for this conception presented by Sheldon Smith (2003), and make it clear that the limit of average velocities is a *limit property* in the sense that it is a property of infinitely many temporal instants (or a relation among them) even though each of those instants except one of them is inessential in determining the property (Section 3).

My third aim is to illustrate how the debate between velocity reductionism and primitivism matters to our understandings of some important issues in physics and philosophy. The discussion on the causal and explanatory roles of velocity is one instance illustrating how the debate matters in physics. In addition, I discuss briefly the overall philosophical project of Descartes to illustrate that the debate also has important implications on other issues in philosophy (Section 2). These examples may not be enough, I admit, to demonstrate the deep and fundamental significance of the debate on physics and philosophy, but I hope that they are enough to get the reader interested in the issue.

2 Reductionism, Primitivism, and Why the Debate Matters

The first clear and precise formulation of velocity reductionism and its justification appear in Bertrand Russell's *Principles of Mathematics*, as follows.

> Motion is the occupation, by one entity, of a continuous series of places at a continuous series of times. … Motion consists *merely* in the occupation of different places at different times, subject to continuity explained [earlier]. There is no transition from place to place, no consecutive moment or consecutive position, no such thing as velocity except in the sense of a real number which is the limit of a certain of quotients. This rejection of velocity and acceleration as physical facts (i.e., as properties belonging *at each instant* to a moving point, and not merely real numbers expressing limits of certain ratios) involves … some difficulties in the statement of the laws of motion; but the reform introduced by Weierstrass in the infinitesimal calculus has rendered this rejection imperative (Russell 1964, 469, 473).

The first two sentences of the above passage constitute an answer to the famous question by Zeno: How can an arrow be said to be in motion in a given temporal interval when at every instant on that interval it only occupies one position and, thus, does not move at that instant? Russell's answer is simple: Motion does not consist of some intrinsic properties of an object at an instant; instead motion is merely the occupation of different positions at different times.

For a proper understanding of the rest of the passage, we need clear definitions of some key terms. A body's *average velocity over a closed temporal interval* $[t_1, t_2]$ is the ratio $(x(t_2) - x(t_1))/(t_2 - t_1)$, where x(t) denotes the body's position at time t. Let v(t) denote the *limit of average velocities* of the body *at* t, as follows:

$$v(t) = \lim_{\Delta t \to 0} [x(t+\Delta t) - x(t)]/\Delta t.$$

And the *limit of average accelerations* a(t) of the body *at* t is:

$$a(t) = \lim_{\Delta t \to 0} [v(t+\Delta t) - v(t)]/\Delta t.$$

Now note that Russell is not denying but acknowledging that a body has the limit of average velocities and the limit of average accelerations at each instant in a given temporal interval when it moves on a smooth trajectory. Russell's rejection of velocity and acceleration as physical facts is the rejection of the limit of average velocities and the limit of average accelerations as instantaneous properties belonging *at an instant* to a body. The reason for this seems to be clear: An average velocity is not a property belonging at an instant to a body but a relation among the body's positions at different instants, for it is defined in terms of two positions at two different instants. Since the limit of average velocities is defined in terms of average velocities, which are relations, it is not a property belonging at an instant to a body but a relation among different positions at various different instants. And the same reasoning applies to the limit of average accelerations.

Futhermore, Russell is claiming that motion involves no other physical properties belonging at an instant to a body than the body's taking up different positions at different instants. In other words, for Russell there is nothing that can be legitimately called 'instantaneous velocity' that is truly instantaneous. What physicists usually call 'instantaneous velocity' is at best a limit of average velocities, which is not a property belonging at an instant to a body but a relation among the body's positions at various moments.

On the other hand, the essence of velocity primitivism appears in impetus theory presented by Buridan in the fourteenth century, as follows.

> [T]he motor in moving a moving body impresses *(imprimit)* in it a certain impetus *(impetus)* or a certain motive force *(vis motiva)* of the moving body, [which impetus acts] in the direction toward which the mover was moving the moving body … And by the amount the motor moves that moving body more swiftly, by the same amount it will impress in it a stronger impetus. It is by that impetus that the stone is moved after the projector ceases to move (Clagett 1961, 534 - 5).

According to Buridan, thus, a body moves when it has a certain amount of impetus, which he defines as the product of weight and velocity. And this velocity must be truly instantaneous, since impetus is an intrinsic property of a body belonging at an instant to the body, irreducible to the body's positions at different times.

Also, Buridan provides explanations, which look very modern, of various observed phenomena using his notion of impetus, such as explanations of why a stone can be thrown farther than a feather, and even why the natural motion of a heavy body downward is continually accelerated (see ibid., 535ff). It is these causal and explanatory roles of impetus and instantaneous velocity that the advocates of velocity primitivism present for the justification of their view—this issue will be discussed in Section 4.

Now the key differences between reductionism and primitivism are as follows. Velocity reductionism says that regarding motion there is no more than a body taking up various positions at different instants. Velocity primitivism disagrees: There is a further intrinsic, instantaneous, and primitive property over and above a body taking up different positions at different times, namely, the property of the body having a certain value of instantaneous velocity at each instant. When a body moves on a smooth trajectory, the instantaneous velocity at an instant coincides with the limit of average velocities at that instant (in our actual world). Yet this coincidence is due to physical laws, not a logical connection. Thus, it is logically possible for the two to have different values and even possible for a body to have instantaneous velocity without having the limit of average velocities. And this instantaneous velocity is an intrinsic property of a body at a given instant because the body's having the instantaneous velocity at that instant is totally logically independent of the body's having whatever intrinsic properties at other instants and also of other bodies' having whatever intrinsic properties at any instant.

Now an often neglected but crucial question arises: how significant is the debate between these two views? I believe that it is fundamentally important both in philosophy and in physics. The problem of change is one of the oldest and most fundamental problems in philosophy, and these two views represent two profoundly different ways of understanding it: According to reductionism change is merely a thing having different properties at different times, whereas on primitivism change is having an intrinsic property of change at an instant. Thus a proper understanding of change requires a proper resolution of the debate between reductionism and primitivism.

Furthermore, the debate is significant even for other philosophical issues. For example, let me briefly discuss the central project of one of the most important figures in modern philosophy, Descartes. Throughout his life Descartes' main philosophical goal was to give a rational explanation for all the phenomena of the material world in terms of the most basic properties of matter. Hence, the famous assertions: "Give me *extension* and *movement* and I will reconstruct the world" (Scott 1952, 161), "the entire universe is a machine in which everything is made of *figure* and *movement*" (ibid.), and "the forms of inanimate bodies … can be explained without the need of supposing for that purpose anything in their matter other than the *motion*, *size*, *shape*, and *arrangement* of its parts" (Descartes 1985, 89).

A modern reader may instantly challenge Descartes' view, pointing out that one of the intrinsic properties of matter, namely, mass, cannot be defined or explained purely in terms of extension, shape, size, arrangement, and motion. But it took several decades after Descartes' death for the concept of mass to be established precisely, and for Descartes the measure of the amount of matter was its volume. Thus, it must be granted that Descartes' view is not internally inconsistent at least in this respect.

With regard to the concept of motion, however, there is a seed of internal inconsistency in Descartes' view. The material world of Descartes consists of nothing but matter, and the sole principal property of matter is extension. As Descartes says in his *Principles of Philosophy*:

> [E]ach substance has one principal property that constitutes its nature and essence, all its other properties being special cases of that. (1) The nature of corporeal substance is extension in length, breadth and depth and any other property a body has presupposes extension as merely a special case of it. For example, we can't make sense of shape except in an extended thing, or of motion except in an extended space. ⋯ But we can make sense of extension without bringing in shape or movement ⋯ (Part I, §53).

Thus, the principal property of a material body is its extension, and all other properties are special cases of that. For example, size or volume is the measure of the amount of a body's extension, shape and figure are determined solely by how the extension of a body is, and arrangement is nothing other than where various bodies and their parts are.

But what about motion? It seems clear that if motion is to be "not a substance but merely a mode of a substance, a way of being that the substance has," which is a special case of extension, then motion must be nothing more than a body taking up different positions at different times (ibid., Part II, §36). That is, it seems that Descartes' motion cannot be an intrinsic property of a body at a given time which is logically independent of the body's extension or location. Descartes' program is, therefore, consistent with velocity reductionism but inconsistent with velocity primitivism. Unfortunately, Descartes has never addressed this issue[1] and has made remarks seemingly contradictory to velocity reductionism,

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[1] This is not an anachronistic comment. In the late Middle Ages, there was a debate between two different conceptions on change and motion, *forma fluens* and *fluxus formae*, which can be considered as the predecessors of modern velocity reductionism and primitivism. For an exposition of this debate, see Dijksterhuis 1961, 174 - 6.

such as: "motion is simply a mode of the matter that moves; but it does have a *definite quantity or amount: how much motion a body has at a given time* is the product of its speed and its size" (ibid., Part II, §36). If Descartes endorsed velocity reductionism at least implicitly, then how do we make sense of 'the definite quantity or amount of motion of a body at a given time'?

Indeed many commentators of Descartes have pointed out that Descartes' concept of the power of a moving body "to persist in its motion, i.e. to continue to move with the same speed and in the same direction," which is partly determined by the body's speed, is inconsistent with his program, for such a power cannot be explained solely in terms of the body's extension (ibid., Part II, §43).[2] Yet note that the problem I raise is new and more fundamental. These commentators have never questioned the status of motion as something solely based on and explicable by extension, but this is one of the key issues that needs to be resolved even before discussing the status of Descartes' forces and powers. There are various attempts to reconcile Descartes' notions of forces and powers with his view that extension is the only principal property of matter and all other properties are special cases of extension, but no serious attempts have been made to examine whether the very notion of motion of Descartes is consistent with his other views.

And Descartes' case is not an exception. There is a huge amount of literature on how Galileo, Kepler, Huygens, Newton, Leibniz, and others have attempted to explain the nature of motion, but there are no explicit discussions on how the debate between velocity reductionism and primitivism matters to their explanations. And

[2] For an exposition of this problem and further references, see Garber 1992, esp. Chapter 9.

this is not because the debate between the two views does not matter: As we have seen, Descartes' whole project is untenable if velocity primitivism is true. I believe that we can learn a lot by reviewing the history of mechanics with the debate between the two views in mind, which I expect to be demonstrated by further studies on these issues.

And it is not just history. The debate between the two views is relevant to our modern understanding of some important features of physics. One example illustrating this point is the topic of the next section.

3. Is the Limit of Average Velocities Really Instantaneous?

At the end of the passage quoted in the beginning of Section 2, Russell says that "the reform introduced by Weierstrass in the infinitesimal calculus has rendered this rejection [of velocity and acceleration as physical facts] imperative." Despite the universal acceptance of Weierstrass' definition of limits nowadays, however, the practice of calling the limit of average velocities 'instantaneous velocity' is now also universal and the belief that the limit of average velocities is really instantaneous is very strong and widespread among physicists and philosophers.

Among physicists, the issue of whether the limit of average velocities at t is a property belonging at the instant t to a body or a relation among the body's different positions at different instants in the neighborhood of t is seldom, if ever, discussed. But we have good reason to believe that most physicists accept, implicitly and without much reflection, that the limit of average velocities is truly instantaneous. First, almost all physics textbooks define the limit of average velocities at t without any discussions about the nature of the definition of limits. This gives the impression to the reader that the limit of average velocities is truly instantaneous: How can the limit be not a property belonging at t to a body but a relation among the body's positions at different instants and still something defined to be relative to *the one single instant* t?

More importantly, it is the notion of 'instantaneous' state that makes it clear that almost all the physicists believe in the instantaneity of the limit of average velocities. In classical mechanics, what physicists call the 'instantaneous' state of a moving body at a given instant t is defined as the pair of its position and momentum at t and is generally understood as a truly instantaneous property that the body has at t. And almost all physicists equate momentum with mass multiplied by the limit of average velocities at t. And if the 'instantaneous' state defined as the pair of position and momentum is a truly instantaneous property belonging at t to a body and if momentum is mass multiplied by the limit of average velocities at t, then the limit of average velocities at t must also be a truly instantaneous property belonging at t to the body. It seems, therefore, that most physicists take the limit of average velocities to be truly instantaneous. Note, however, that this does not establish that the limit of average velocities is truly instantaneous, for the claim that momentum or 'instantaneous' state is truly instantaneous needs to be justified first.

For philosophers, opinions are diverse. Sheldon Smith (2003) presents most notable arguments for the belief in the instantaneity of the limit of average velocities, as follows—note that, in the following quote, the interval (a, b) denotes the open interval which includes neither a nor b, and that by velocity at t Smith means the limit of average velocities at t:

> [V]elocity is a property of an instant t alone because … one cannot identify precisely what non-t points are responsible for the implication that … the velocity is a property of those points in addition to t. … If we know the state of the particle only within some set of points around t such as $(t-\delta, t-\varepsilon) \cup (t+\varepsilon, t+\delta)$ where $\varepsilon>0$ is less than $\delta>0$, that will tell us nothing about the velocity at t. One can say unambiguously that the implication about the velocity at t is *not* arising from that set of points since they do not have the implication at all. Thus, it can be said that the velocity at t is *not* a property of what is going on at those points. …
> We cannot say what set of non-t points is responsible for that implication for the velocity at t because we can always move ε in towards t in a way that it captures any point, and $(t-\delta, t-\varepsilon) \cup (t+\varepsilon, t+\delta)$ where $\varepsilon>0$ is less than $\delta>0$ *never* carries the implication. … We never get an answer to the question: What other than t is the velocity a property of? This suggests to me that it is just a property of t alone (275-6).

Smith's argument in the above passage can be reconstructed as follows:

Argument I

Premise 1: For any real numbers δ and ε such that $\delta>\varepsilon>0$, the body's trajectory on the interval $(t-\delta, t-\varepsilon) \cup (t+\varepsilon, t+\delta)$ has no implication on the value of the limit of average velocities of the body at t.

Premise 2: For any real numbers δ and ε such that $\delta>\varepsilon>0$, if the body's trajectory on the interval $(t-\delta, t-\varepsilon) \cup (t+\varepsilon, t+\delta)$ has no implication on the value of the limit of average velocities of the body at t, then the limit of average velocities of the body at t cannot be a property of the body's trajectory on $(t-\delta, t-\varepsilon) \cup (t+\varepsilon, t+\delta)$.

Conclusion: The limit of average velocities of the body at t is just a property of t alone.

This is an invalid argument. To see this point, consider an infinite sequence $a_1, a_2, \cdots, a_n, \cdots$, which converges to a. Of this sequence one can legitimately say that the sequence $a_1, a_2, \cdots, a_n, \cdots$ possesses the property of having a as its limit. But no one (or no finite subcollection) of $a_1, a_2, \cdots, a_n, \cdots$ is necessary to define the limit. That is, for every n, $a_1, a_2, \cdots, a_n$ have no implications on the value of the limit of the sequence. Now consider the following argument.

Argument I*
Premise 1*: For every n, the finite subcollection $A_n=\{a_1, a_2, \cdots, a_n\}$ has no implication on the value of the limit of the sequence $a_1, a_2, \cdots, a_n, \cdots$.
Premise 2*: For every n, if A_n has no implication on the value of the limit of the sequence, then having a as the limit is not a property of A_n.
Conclusion*: Having a as the limit is just a property of the null set.

This is an invalid argument since the conclusion is evidently false though the premises are true. From Premises 1* and 2*, it follows that for every n, having a as the limit is not a property of $A_n=\{a_1, a_2, \cdots, a_n\}$. But this does not mean that having a as the limit is not a property of the infinite sequence $a_1, a_2, \cdots, a_n, \cdots$ nor that having a as the limit is just a property of the null set.

Likewise, even if Premises 1 and 2 of Argument I are true, its conclusion does not follow. From Premises 1 and 2, it follows that for any real numbers δ and ε such that $\delta>\varepsilon>0$, the limit of average velocities of the body at t cannot be a property of the body's trajectory on $(t-\delta, t) \cup (t, t+\delta)$. But this never means that the limit of average velocities of the body at t cannot be a property of the body's trajectory on (t−δ, t+δ) nor that it is just a property of t alone.

Then, what is the limit of average velocities at t a property of? The limit of average velocities has several puzzling features, all of which are due to the nature of infinity. And one of them is the following: It is evident that having a as the limit is a property of the infinite sequence a_1, a_2, ⋯, a_n, ⋯ even though no single member of the sequence is essential in defining the limit. Thus, having a as the limit is some kind of a collective property of the infinitely many numbers a_1, a_2, ⋯, a_n, ⋯, even though the amount of contribution each single individual member makes to the collection possessing the property is null or infinitesimal. There are a bunch of examples of such collective properties when infinities are involved−see Hawthorne 2000 and Yi 2008 for some recent discussions on such properties.

Thus we should not deny that the limit of average velocities at t is a property of the body's trajectory on any given interval surrounding t even though no point on that trajectory except the body's position at t is essential in determining the limit at t. Contrary to what Smith says, therefore, we get an answer (and actually many good answers) to the question: What other than t is the limit of average velocities a property of? The answers are: It is a property of every interval surrounding t.

In short, the limit of average velocities is a limit property: Property P is a *limit property* if it is a property of infinitely many things and yet each of those things except some finite subcollection of them is inessential in determining the property. And besides those ones involving limits, there are many other instances of limit properties. The property of being in motion at an instant, which is defined as follows, is such an example. Almost all people would agree that in order to be able to observe motion there must be two different instants at which a given body occupies two different positions. Similarly a body is not motion just in case its position does not change during some temporal interval. Generalizing these intuitions, let us make the following definition:

> A body is *not in motion at* t if and only if there exists some real number $\varepsilon>0$ such that for every real number $t*$, if $|t*-t|<\varepsilon$ then $x(t*)=x(t)$,

or equivalently,

> a body is *in motion at* t if and only if for every real number $\varepsilon>0$ there exists some real number $t*$ such that $|t*-t|<\varepsilon$ and $x(t*)\neq x(t)$.

According to this definition, a body is in motion at t only if there exists some temporal instant t* distinct from t such that the body exists at t*. That is, a body being in motion at t implies it existing at some instant other than t. Thus, being in motion cannot be an instantaneous property belonging at t to the body. Instead, being in motion at t is a property of an interval surrounding t. But no part of any interval surrounding t except for t is essential in determining whether or not a given body is motion at t. Thus being in motion at t is a limit property—note that the property of being in motion at t is different from the property of having a nonzero limit of average velocities at t since it is possible for a body to be in motion at t while the limit of average velocities at t is 0.[3]

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[3] For example, at the moment at which a projectile thrown upward reaches its maximum height, the limit of average velocities is 0 even though it is in motion at that moment according to the definition.

It is, therefore, wrong to take the limit of average velocities at t as a truly instantaneous property belonging at t to a body. And this conclusion gives rise to a serious challenge to the common practice of physicists: We have seen that almost all physicists take the 'instantaneous' state in classical mechanics to be defined as the pair of position and momentum at a given instant t, and they also take momentum at t to be defined as the product of mass and the limit of average velocities at t. Now, if the limit of average velocities at t cannot be truly instantaneous, then what physicists call the 'instantaneous' state cannot be truly instantaneous, either. If physicists want the state to be truly instantaneous, they should abandon the practice of defining momentum as the product of mass and the limit of average velocities at t but adopt velocity primitivism and define momentum as the product of mass and the body's intrinsic instantaneous velocity at t. Most physicists, however, would not be willing to do this, believing velocity primitivism to be unnecessary and too metaphysical.

Contrary to the view popular among physicists, David Albert (2000) claims that the limit of average velocities should not be counted as part of the instantaneous physical state of the world at a given moment: "[A] specification of the positions and the [limits of average] velocities of all the particles in the world at some particular instant is *not* a specification of the physical situation of the world at that instant *alone*; it is *not* a specification of the physical situation of the world at that instant as *opposed to all others*, at all" (10 - 11).

Of course Albert is not saying that knowledge of the positions of all the particles in the world at a given moment is enough for predicting their positions at other times. Albert calls the full predictive resources of the dynamical laws of physics *dynamical* *conditions*, which include position and limit of average velocities, and admits that dynamical conditions can be in one way or another uniquely attached to an instant. But again Albert adamantly claims that dynamical conditions are not a description of the world at a given instant as opposed to all others.

This difference is not a merely verbal one. It does matter to our understanding of various important features of physics. For example, by taking the instantaneous state at t to be defined solely in terms of the positions of the given bodies at t and denying the instantaneity of the dynamical conditions defined as the pair of the positions and momenta of the bodies at t, Albert (2000, chapter 1) constructs a very convincing argument that classical electrodynamics is not invariant under time-reversal whereas classical mechanics is—this is a very interesting point but our focus is elsewhere. What is directly relevant to our topic is that there is a powerful argument that if neither the limit of average velocities nor what physicists call the 'instantaneous' state are truly instantaneous, then classical mechanics (nor relativity theories) cannot actually provide some causal explanations which many physicists and philosophers have uncritically believed it can, namely, the kind of causal explanations of the position of a particle at t^* in terms of its position at t and the forces which act on the particle during the interval between t and t^* ($t^*>t$). That is, the challenge is that if we adopt velocity reductionism then we should give up both the idea of truly instantaneous states and the belief that classical mechanics can provide genuine causal explanations.

Advocates of velocity reductionism may just concede that classical mechanics does not aim for nor actually provide causal explanations, as Russell did—see Russell 1917, Chap. 9. Yet what is at issue is whether the adoption of velocity reductionism *requires* us to give up causal explanations in classical mechanics. There is no general agreement that classical physics must dispense with causal explanations. So if it is viable to provide causal explanations in classical mechanics and velocity reductionism requires us to give up causal explanations, then it is a bad news for velocity reductionism. Whether velocity reductionism has such far-reaching consequences will be discussed in the next section.

4. The Causal Explanation Problem and Its Resolution

One of the most powerful arguments raised against velocity reductionism is that if reductionism is right then velocity cannot play the causal and explanatory roles traditionally ascribed to them in classical physics. This argument is presented in the clearest form by Marc Lange (2005, 2009) as follows—note that '$(t_0, t_0+\Delta t]$' denotes the half-open half-closed interval which includes $t_0+\Delta t$ but not t_0; $[t_0, t_0+\Delta t]$ is the closed interval which includes both t_0 and $t_0+\Delta t$:

> What does a body's instantaneous velocity bring about? It figures in causal explanations of the body's subsequent trajectory. In accordance with Newton's second law of motion, a body's trajectory in the interval $(t_0, t_0+\Delta t]$ can be causally explained by the body's mass, the forces on the body at each moment in the interval $[t_0, t_0+\Delta t]$, and some initial conditions: the body's position at t_0 and (here comes our concern) the body's velocity at t_0. … But … the body's $v(t_0)$ is a cause, under the reductive interpretation, only if the body's trajectory in a neighborhood of t_0 is a cause—and any such neighborhood includes moments after t_0. Hence, in order for $v(t_0)$ to be a cause of the body's trajectory in $(t_0, t_0+\Delta t]$, the body's trajectory in $(t_0, t_0+\Delta t]$ would have to be a cause of itself. This

> cannot be. Here we have the germ of a powerful argument against velocity reductionism: that it cannot account for velocity's causal and explanatory role (2005, 438).

Lange calls this problem the *causal explanation problem*. One can avoid the causal explanation problem by maintaining that classical mechanics should not be interpreted causally. Yet even if it is overall better to interpret classical mechanics non-causally than causally, it is worthwhile to investigate whether reductionism is really incompatible with such causal and explanatory explanations in classical mechanics. Thus, I accept this challenge of providing causal explanations in classical mechanics and argue that reductionism can meet this challenge. I will argue that the kind of causal explanation Lange demands in his formulation of the causal explanation problem is not the right sort of explanations which we are supposed to provide in classical mechanics, and that reductionism admits of the right sort of explanations.

First let us make clear what the explanans (or causes) and explanandum (effect) of the causal explanation demanded by Lange are: The explanans are (1) the forces on the body at each moment in the interval $[t_0, t_0+\Delta t]$, (2) the body's mass, (3) the body's position at t_0, and (4) the body's velocity at t_0; the explanandum is the body's trajectory in the interval $(t_0, t_0+\Delta t]$. And Lange's argument is that since reductionism identifies the body's velocity at t_0 with the limit of average velocities at t_0, which is a property of the body's trajectory on some interval surrounding t_0, reductionism renders the body's trajectory around t_0 a cause of the trajectory on the interval $(t_0, t_0+\Delta t]$, thus, making some portion of the body's trajectory on $(t_0, t_0+\Delta t]$ a cause of itself, which is problematic. In short, the causal explanation problem arises because one of the explanans, namely, (4), which is identical with a property of the body's trajectory on $(t_0, t_0+\Delta t]$ on reductionism, cannot legitimately explain the explanandum, which is the body's trajectory on the same interval.

When the causal explanation problem is formulated this way, however, it cannot be solved even if we give up reductionism and accept primitivism, for the problem also lies in (1). To see this, consider the following situation which Lange (2005, 448) himself discusses: A charged body is moving in an electric field without feeling any other forces. The body's acceleration a(t) is caused by its mass and the electric force it feels. That force F(t), in turn, is caused (in accordance with $F = qE$) by the field E at a given location, along with the body's possessing charge q and occupying that location x(t) at t. So, the body's positions on the interval $(t_0, t_0+\Delta t]$ are causes of the forces on the body on the same interval. Now note that causation is transitive (that is, if A is a cause of B and B is a cause of C, then A is a cause of C). Thus if Lange's formulation of the causal explanation problem is legitimate, then the forces on the body at each moment in the interval $[t_0, t_0+\Delta t]$ are causes of the body's trajectory in the interval $(t_0, t_0+\Delta t]$, and so it follows that the body's positions on the interval $(t_0, t_0+\Delta t]$ are causes of themselves, which is problematic.

This problem arises because Lange's formulation of the causal explanation problem is not fine enough: The forces on the body in any interval (t_1, t_2) cannot explain the body's positions on any earlier interval (t_0, t_1) at all, and yet in Lange's formulation, they appear as a common part of the explanans and explanandum.

One way to avoid this problem is to reformulate the causal explanation demanded in classical mechanics as follows:

For all instants t_0 and t such that $t_0<t$, the body's position at t

> should be causally explained by the body's mass, the body's position and velocity at t_0, and the forces on the body on the interval $[t_0, t)$.

This scheme of causal explanation does not contain any element of self-cause. All the things in the explanans are events which happen before t and the explanandum is an event happening at t: The limit of average velocities at t* can certainly be identified as a property of some portion of the body's trajectory before t.

Lange actually considers explanations like this. He says, "for any point x(T) of that trajectory, there is a causal explanation with one initial condition consisting of a certain relation's holding among the points in the body's trajectory in any neighborhood around t_0 that is small enough to exclude T" (2005, 440 - 1). Yet Lange rejects this explanation for the following reason:

> However, an important aspect of $v(t_0)$'s causal role in classical physics is that it serves as a cause of all of the points in the body's trajectory in $(t_0, t_0+\Delta t]$. It is a common cause of the body's position at every later moment—no matter how remote from t_0, and certainly no matter how near. On the above proposal, the common cause we find in $v(t_0)$, a relation's holding among the points in the body's trajectory, is lost when we proceed to take the relata as causes. For any two points $x(t_0+\Delta t)$ and $x(t_0+\delta t)$ in the body's trajectory after t_0 (indeed, for any finite number of points), there is a neighborhood around t_0 where all of the points in the body's trajectory in that neighborhood, fixing $v(t_0)$, can serve as common causes of $x(t_0+\Delta t)$ and $x(t_0+\delta t)$. But no single neighborhood can play this common-causal role for all of the points in the body's trajectory in $(t_0, t_0+\Delta t]$. Thus, velocity reductionism fails to respect a slight extension of the principle we presupposed (that a relation's holding is a cause only if the relata are)—namely, that a relation's holding is a common cause only if the relata are (Lange 2005, 441).

This is a brilliant point, which reveals another puzzling feature of a limit property. For every point t in (t_0, $t_0+\Delta t$], there exists a neighborhood around t_0 but before t which can fix the body's velocity $v(t_0)$, and so $v(t_0)$ is a common cause of all of the points in the trajectory in (t_0, $t_0+\Delta t$]. There does not exist, however, a neighborhood around t_0 which is before t for every t in (t_0, $t_0+\Delta t$]. That is, there is no neighborhood around t_0 which can play the role of a common cause of all of the points in the trajectory on (t_0, $t_0+\Delta t$].

Contrary to Lange's conclusion, this does not mean that reductionism cannot accomodate the intuition that the body's velocity at t_0 is a common cause of all of the points in the trajectory on (t_0, $t_0+\Delta t$]. To see this, let us first make clear what it means to say 'the limit of average velocities $v(t_0)$ is a cause of the body's position at t_1' ($t_0<t_1$). Since $v(t_0)$ is merely a property, '$v(t_0)$ is a cause' is a shorthand expression for '(the event of) some thing(s) possessing the property $v(t_0)$ is a cause.' On reductionism, the things that can possess the property $v(t_0)$ are the body's trajectory on any interval surrounding t_0. But as we have seen, the limit of average velocities $v(t_0)$ is a limit property, and so we cannot choose one particular interval and then identify $v(t_0)$ with the property of the trajectory on that interval. Thus, we should not associate the limit of average velocities $v(t_0)$ with the trajectory on any one particular interval around t_0.

This means that when we say '$v(t_0)$ is a cause of the body's position at t_1,' we mean 'there exists some (unspecified) interval around t_0 such that the body's trajectory on that interval having the property $v(t_0)$ is a cause of the body's position at t_1.' And '$v(t_0)$ is a cause of the body's position at t_1 and also a cause of the body's position at t_2' ($t_0<t_2$) does not mean 'there exists some

interval around t_0 such that the body's trajectory on that interval having the property $v(t_0)$ is a cause of the body's position at t_1 and also a cause of the body's position at t_2,' but means 'there exists some interval around t_0 such that the body's trajectory on that interval having the property $v(t_0)$ is a cause of the body's position at t_1 and there exists some (possibly different) interval around t_0 such that the body's trajectory on that interval having the property $v(t_0)$ is a cause of the body's position at t_2.' Likewise, '$v(t_0)$ is a common cause of all of the points in the trajectory in $(t_0, t_0+\Delta t]$' does not mean 'there exists an interval around t_0 such that the body's trajectory on that interval having the property $v(t_0)$ is such a common cause.' Rather it means 'for every t in $(t_0, t_0+\Delta t]$, there exists an interval around t_0 such that the body's trajectory on that interval having the property $v(t_0)$ is a cause of the body's position at t'. In short, the claim, according to reductionism, that the body's velocity at t_0 is a common cause of all of the points in the trajectory in $(t_0, t_0+\Delta t]$ is nothing more than that for any t in $(t_0, t_0+\Delta t]$, the body's position at t can be causally explained by what is happening in (t_0, t).

In this way, reductionism can accommodate the intuition that $v(t_0)$ is a common cause of all of the points in the trajectory in $(t_0, t_0+\Delta t]$. So there is nothing essentially satisfying or kinematically compelling in the attempt to posit velocity as something which should be identified to be some intrinsic or dispositional property possessed by the body. That is, you do not gain any explanatory power by giving up reductionism and taking these alternatives instead. Though there is no common interval that can serve as a common cause, the fact that those intervals all share a common property can justify saying that the body's velocity at t_0 is a common cause. Therefore velocity's causal and explanatory roles can be defended even under reductionism.

5. Conclusion

I have argued that, contrary to common beliefs of many physicists and philosophers, the limit of average velocities is not truly instantaneous, but that it can play the causal and explanatory roles that classical physics is often interpreted as demanding of them. These points suggest that it is a viable metaphysical option to identify what many physicists call 'instantaneous' velocity with the limit of average velocities, but doing so has many consequences and implications, besides those discussed in Sections 2 and 3, which would surprise those who mistake the limit of average velocities to be truly instantaneous. What those consequences and implications will be are left for further investigation.

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