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An Epistemic View of Mathematical Explanation and Its Ontological Import

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Some mathematical realists attempt to establish that there are legitimate mathematical explanations of empirical phenomena. They argue that if there are such explanations, and if they are our best explanations of relevant phenomena, we ought to be ontologically committed to mathematical facts or objects. Expectably, some mathematical nominalists reject the legitimacy of mathematical explanation. In this paper, I argue that this entire debate is based upon a unsustainable or narrow conception of explanation, the ontic conception of explanation. This leads us to the conclusion that, when we adopt an epistemic conception of explanation, accommodating legitimate mathematical explanations no longer necessitates our ontological commitment to mathematical facts or objects.

Keywords: Scientific Explanation, Epistemic concepts of explanation, Mathematical explanation, Mathematical Realism, Indispensability argument

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1. Introduction

A mathematical explanation of an empirical phenomenon (hereafter, 'mathematical explanation') is *said to be* an explanation that includes at least one mathematical fact within the explanans, and an empirical phenomenon as the explanandum. According to this definition of 'mathematical explanation', adopting any alleged mathematical explanation as a legitimate scientific explanation carries significant ontological implications. That is, asserting the legitimacy of mathematical explanations implies the existence of a certain explanatory relation, with its relata encompassing a mathematical fact.

(The term 'legitimacy' is used in this context in a somewhat abstract and ambiguous manner. It can be loosely likened to concepts such as explanatoriness or success, which are related to assessment. While the intent is not to provide an exhaustive definition of 'legitimacy' or to dictate how explanations should be evaluated, this paper seeks to delineate what it is not. Essentially, the term 'legitimacy' serves as a broad foundation for assessing explanations.)

Unsurprisingly, the question of whether there are legitimate mathematical explanations has garnered attention from some philosophers of mathematics, especially who are engaged in the mathematical realism/nominalism debate. Of course, it is not the case that those who accept the legitimacy of mathematical explanations always support mathematical realism. At least, some of them explicitly put themselves in a neutral position.[1)] Nor is it that all those who refuse the existence of mathematical facts always reject the legitimacy. As I hinted in the first paragraph, the definition is somewhat arbitrary. They may be able to accept the legitimacy while taking an alternative definition of 'mathematical explanation' which

1) e.g., Berkovitz (2019), p. 34.

does not interrupt their own ontological view.

But some of them do tie the two issues. On one the one hand, certain mathematical realists have endorsed the thought that alleged mathematical explanations are indeed legitimate scientific explanations (Baker 2005; Colyvan 2010). As a result, they argue that we ought be ontologically committed to mathematical facts or objects exploited in such explanations, since those cases may exemplify indispensability of mathematical facts to our best theory. On the other hand, some mathematical nominalists have explicitly refused the legitimacy of mathematical explanations (Bueno 2012; Strevens 2018; Barrantes 2020). They attempt to show, or presume that these explanations are, in fact, disguised empirical explanations that only involve concrete facts as their relata, whether in causal or non-causal contexts.

In this paper, I argue that the aforementioned approaches to understanding the conception of mathematical explanations are misguided. I will argue that those approaches presume a unsustainable conception of explanation, which is also presupposed in the earlier definition of mathematical explanation. When we adopt the proper conception of explanation, the thought that there are some legitimate mathematical explanations no longer necessitates the existence of mathematical facts or objects.

The unacceptability of the underlying conception of explanation will be demonstrated by following the debate between the ontic and epistemic views of explanation. In section 2, I will introduce and refine the challenges against the ontic view, the view that I purpose to reject. This will lead us to a conclusion about how explanations should *not* be assessed. I will call this conclusion as 'the deregulation.' In section 3, based upon the deregulation, I will argue how a legitimate mathematical explanation can be exempted from the need for ontological commitment to mathematical facts or objects. Finally, in section 4, I will summarize the entire discussion and give rise to some general remarks.

2. The ontic conception of explanation and its problem

Debates surrounding the notion of scientific explanation have a conceptual question. That is, what is the conception of explanation at all? There are two prominent viewpoints on this matter. A tradition in the philosophy of science, coupled with our pre-theoretic intuition of explanation, posits that an explanation is primarily an epistemic, communicative, or linguistic practice. Hence, a faction of philosophers views the items of explanation as knowledge, information, or expressions, rather than as worldly facts. In that sense, this view is called as '*the epistemic view of explanation.*' According to this view, the structure of an explanation is best understood as *a complex of representations*.

Opponents of this view argue that there are explanatory relations that hold among worldly facts or events, regardless of the presence of any cognitive agent who can recognize such relations. On this view, the notion of explanation is ontic. So we call this view as '*the ontic view of explanation*.' From an ontic standpoint, an explanation is something that may or may not be obtained, but it lacks a truth or falsehood dimension. And, in contrast to the epistemic view, the structure of an explanation is seen as *a complex of factive entities*.

In the debate between ontic and epistemic views, the burden of proof predominantly falls on the ontic view. Because our pre-theoretic linguistic intuition about the term 'explanation' is associated with certain kinds of communicative acts that convey information. In the tradition of the philosophy of science, several philosophers have explicitly endorsed the thought that the conception of explanation is closely related to the concepts of understanding, learning, and knowledge. The entry by Wright and van Eck (2018), in which themselves advocate the epistemic view,

nicely summarizes the perspectives of various philosophers of science.[2] According to them, these philosophers have regarded the matter of explanation as belonging to the realm of epistemology (Humphreys 1989; Ruben 1990; Faye 1999), and they see explanation itself as a form of epistemic activity (Lycan 2005; Bechtel 2008).

But, then, what does motivate the opponents? While there may not be a single overarching motivating factor, their starting point is clear in contexts of justification. Proponents of the ontic view set out by observing the equivocation of the term 'explanation' or 'explain' in English. For instance, while it seems legitimate to say "Alice explained the reason that she was late to the audience", it is also acceptable to say, "the gravitational attraction of the moon explains the tides."[3] In the former statement, the verb 'explain' signifies a specific speech act performed by Alice. In contrast, the (seemingly) same verb in the latter statement appears to indicate a relation held between the gravitational attraction of the moon as an event and the tides as another. Proponents of the ontic view take this observation of equivocation as evidence that the explanation-related term have multiple senses. And, as seen in the instance above, one of these senses is the ontic sense referring to a relation between factive entities in the outer world.

Given the legitimacy of the ontic sense of the term 'explanation', proponents of the ontic view take a further step to assert that the "actual" conception of explanation should the ontic conception, while "[t]he linguistic entities that are often called 'explanation' are statements reporting on the actual explanation."[4] This position distinguishes explanatory representations (e.g. texts, speech acts, subpersonal physiological

2) Wright and van Eck (2018), pp. 999–1000.
3) Salmon (1989), p. 86.
4) *Ibid.*, p. 133.

mechanisms, and so on) from what a use of 'explanation' should actually or basically mean. So, strictly speaking, such representations are *not* explanations.

Opponents complain that "the only things that could ultimately justify [this position] is an ideological commitment to [the ontic conception]." So, there is no good reason to take this position without question-begging.[5] This prompted proponents of the ontic view to turn to the claim of "normative priority" of the ontic sense over epistemic ones. In this move, they admit that there are different meanings of 'explanation', which amount to the ontic and epistemic senses, and advise caution, saying: "It would be a mistake to conflate them."[6] Still, they hold that there is a priority of epistemic senses of explanation. Proponents of the epistemic views may concede that an explanation (conceived in an epistemic sense) could possess a truth or false dimension; though I will demonstrate that they need not necessarily do so. But how can it be assessed whether a given explanation is true or false? It may depend on whether or not the explanation represents a real explanatory relation held between some relevant factive entities. In contrast, according to the ontic view, as a use of 'explanation' in the ontic sense refers to a relation in the world, "[explanations] are full-bodied things. They are not true or false."[7] Hence, an ontic explanation is inherently explanatory, while representations are considered true or successful explanations only when they correspond to ontic explanations. Thus, it seems plausible to assert that "there is a kind of *fundamentality* associated with [explanations conceived in the ontic sense]"[8], since there is an asymmetric direction of normative dependency between ontic and epistemic senses.

5) Wright and van Eck (2018), p. 1016.
6) Craver (2014), p. 35.
7) *Ibid.*, p. 40.
8) Sheredos (2016), p. 927, my emphasis.

I shall examine two prominent objections against the ontic view, which I call as '*No Foundation Objection*' and '*Narrow Conception Objection*', respectively.

No Foundation Objection

Note that proponents of the ontic view heavily rely on their observation of the ambiguity of the term 'explanation' in the English language. They take statements like "the gravitational attraction of the moon explains the tides" at face value. So, from their perspective, the gravitational attraction of the moon *literally* provides an explanation for the tides.

Critics of the ontic view contend that this construal is superficial and overlooks the deeper structure of such statements. They argue that expressions, in which the term 'explanation' or 'explain' appears to indicate an ontic sense, are simply "elliptical figures of speech that, when reinterpreted literally, are not true and do not transparently designate a relation of ontic explanation."[9] Wright, in another work (2012), observes that the term 'explain' fails to pass several ambiguity tests in semantic analysis, which means that the term is *not* truly ambiguous despite surface appearances. The term 'explanation' fails to pass a homonymy test since it fails to adduce a pair of sensible sentences that have the same syntactic structure but distinct semantic structures[10]; it fails to pass the test of contradiction which check a term can occur both truly affirmed and truly

9) Wright and van Eck (2018), p. 1014.

10) For instance, the obvious homonymy 'match' can be used to construct such a pair: 'The entire match could not be {played/lit} due to the rain.' Here, both sentences are grammatically sensible due to the target term's homonymy. In contrast, the term 'explanation' fails to construct such a pair: when we consider the pair of sentences 'The full explanation could not be {stated/caused} due to the lack of evidence', only the first one is grammatically sensible.

denied in a declarative sentence[11]; it cannot construct an ambiguous syllogism; and so on.[12][13]

This result conflicts with the observations emphasized by proponents of the ontic view. What could be the best explanation of this discrepancy? Since Wright (2012) used various testing methods, and the term fails to pass most of them, it is implausible to reject the credibility of all those testing methods and dismiss their results. Rather, as Wright suggests, it is more reasonable to think as that *the supposed ontic conception of explanation is merely metaphorical*. Consider an English expression

11) For instance, sentences like 'That bank is not a bank' are sensible because the term 'bank' is ambiguous. But sentences like 'To explain is to not explain' is not.

12) see Wright (2012), pp. 385-7 for details.

13) To be fair, Wright concedes that there is an important test according to the result of which suggests that the term 'explain' is ambiguous, the syllepsis/conjunction reduce test. Roughly, syllepsis/conjunction takes as an ordered pair of clauses in which the target term is applied to distinct objects which, if the target term is really ambiguous, each of the distinct senses may be applied to. It returns a single clause constructed by their conjunction and reduction. Here again, just citing Wright (2012, pp. 384-5)'s examples, consider the pair of clauses {'Close your door, and then go to sleep', 'Close you eyes, and then go to sleep'}. Syllepsis/conjunction taking the pair as input returns the single clause 'Close your door and eyes, and then go to sleep.' If the resulting clause is judged grammatically acceptable, like this case, the target term fails the sylleptic test and so is not considered as ambiguous. As an opposite example, consider the pair of clauses {'The mistakes were committed', 'The patients were committed'} which is reduced to 'The mistakes and patients were committed' through the syllepsis/conjunction reduction. The resulting clause is grammatically unacceptable, and then the target term 'be committed' is said to pass the test. This implies that "the data counts as defeasible evidence for its being ambiguous." Now, for instance, the grammatical unacceptability of the clause '*The investigator* and *the oxygen* canisters *explained* why the ValuJet crash in the Florida Everglades occurred' appears to indicate that the target term 'explain' is ambiguous.

However, provided with the fact that the same target term fails to pass other several tests, this result can only make the issue more puzzling.

which seemingly contains the ontic conception of explanation, such as the Salmon's example sentence: "the gravitational attraction of the moon explains the tides." Regarding such an expression, it seems not difficult to find a corresponding statement that stands for a circumstance in which a cognitive agent explains (in an epistemic sense) the same phenomenon using *a model of* the explanans. That is, we can understand Salmon's sentence as a metaphorical abbreviation of, for instance, the sentence: "Physicists explain the *presence* of tides *with a physical model of* the gravitational attraction", for the sake of pragmatic simplicity. More generally, Wright and van Eck (2018), after Collins (1966), suggest a translation rule: "if appeal to or use of x is a means by which [a subject] S can get y done, then speaking figuratively we can say, 'x gets y done'."[14)15)]

I concur with this criticism. But it does not appear to be very decisive to me. As Wright concedes, the term 'explain' passes some important ambiguity test (see footnote 13), although it fails to pass others. Insofar as proponents of the epistemic view complain that "the only things that could ultimately justify [the ontic view] is an ideological commitment to [the ontic conception]", it is unclear how firmly we can depend on linguistic evidence. Moreover, Craver (2014) suggests some examples of

14) Wright and van Eck (2018), p. 1012, my italic.

15) Before suggesting this translation rule, their focus is on cases where causation-related terms are used figuratively as explanation-related terms. Thus, it may seem as if this suggestion is appliable only when x is causally efficient to y. But I think this account can be extended to cases of more general terms which are related to alleged non-causal explanatory relations. For instance, in favor of the ontic view, it seems legitimate to say "the atoms arranged such-and-such *explain* the existence of the Eiffel tower." This sentence might be uttered, in appropriate contexts, instead of the less contentious sentence: "the atoms arranged such-and-such *ground* the existence of the Eiffel tower." Nevertheless, we should not be adverse to understand the former as an elliptical or metaphoric form of "Metaphysicians explain the presence of the Eiffel tower appealing to a physical model describing the atoms arranged such-and-such."

uses of 'explanation' which are "awkward or nonsensical" if we tie the conception of explanation to representations.[16] I do not think one cannot regard his examples metaphoric, but there would be no reasoning which is absolutely neutral to any ideological commitment. This kind of dispute may be endless. Hence, although I will contemplate the implication of this uncompromising objection in the subsequent section, what I really take seriously is the next objection.

Narrow Conception Objection

In the entire debate between the ontic view and the epistemic view, the crux of the matter is to preserve our uses of explanation-related terms. For, while the epistemic view appeals to our commonsense of such uses, the ontic view challenges to the commonsense paying attention to another aspect of such uses, which appears to indicate the equivocation of the terms. However, even if the line of argument for the ontic view were correct, proponents of it face an additional problem in that it cannot effectively accommodate somethings we count as explanations in scientific practice.

Scientific explanations contain the norms of abstraction (or, idealization) and generalization. That is, scientific explanations often abstracts away from certain details, and, are generally applied to a group of similar phenomena, even when they do not perfectly fit. The epistemic conception of explanation can readily adopt these norms. According to the epistemic view, an explanation is a complex of representations, and these representations are, indeed, often obtained through processes of abstraction. For instance, a gravitational model for explaining the tides may abstract away from the gravitational force of all celestial bodies other than the moon, which is neglectable due to the distance from the

16) Craver (2014), p. 36.

surface of the Earth. Moreover, a representation has a range of things to which it can be applied, and, perhaps, the range contains a thing that does not perfectly satisfy the conditions specified by the representation. For instance, in optics, *the formula of* Snell's law is exploited to explain some optical phenomena in anisotropic medium, despite the formula itself is to be applied only to phenomena in isotropic medium (see Cartwright 1980).

But how can a proponent of the ontic view construe an abstract and generalized explanation? An ontic explanation is expected to be a particular relation (whether causal or non-causal) between factive entities, which are real entities in the world. This gets into troubles regarding both of the norms. Previous studies in favor of the epistemic view have generally described this issue by noting that abstraction and generalization are epistemic processes, and thus, the ontic view cannot easily accommodate them under its conception of explanation. I hope I can develop this idea more systematically.

Let's begin with the norm of abstraction. Wright and van Eck already pointed out this:

> [A]bstraction is a cognitive process characterized by focal adjustments of deselection. And since the relata of cognitive processes are themselves cognitive structures, it follows that abstracta aren't non-representational and mind-independent ontic structure. (Wright and van Eck 2018, p. 1020)

But this paragraph seems to be too averse to the existence of mind-independent abstract entities. What if a proponent of the ontic view is also a realist about abstract entities of several kinds? Can't she say that an abstract explanation is an ontic relation of which relata are abstract entities?

I think the real issue is that a proponent of the ontic view cannot *freely* take an abstract entity to exist. According to the ontic view, an explanation should refer to a relation of factive entities, so that, if there is a genuine abstract explanation subsumed to the ontic conception, this explanation must refer to a relation among abstract entities. But then, it is required for the proponent to be ontologically committed to such abstract entities. How can she justify that commitment? One might argue against the Quinean criterion, according to which we should be ontologically committed to all and only what our best theory regimented in the first-order language *quantifies over*. Nonetheless, only a few would disagree with the Quinean in that our ontological commitments should altogether be, in some broad sense, guided by our *best* theory. That is, her only rationale to be committed to relevant abstract entities, and to take an abstract explanation to be an ontic relation of them, is to appeal to the bestness of a theory that is committed to those abstract entities.

However, note that the conjunction of (1) that there is a genuine abstract explanation for a phenomenon, and (2) that the abstract explanation is the best one, is not sufficient to justify the existence of the relevant abstract entities. Friends of the epistemic view can also adopt (1) and (2) for the very same explanation, with the translation rule suggested above by Wright and van Eck. For them, the terms 'explanation' in (1) and (2) simply refers to a complex of representations, which does not require any abstract entities to exist in the outer world. Here, the key issue is how to read the explanation in question. Both camps may well agree on which explanation is the best one, except for their interpretations of its commitment at the fundamental level. Hence, what the proponent of the ontic view should offer is an independent argument for that that an abstract entity explains another should be *literally* interpreted, but not as a figure of speech which should be reinterpreted via some translation rules. But claiming that it should be literally interpreted is nothing other than accepting the ontic

view. Consequently, the ontic view of explanation lacks a neutral way of being ontologically committed to abstract entities which are desiderata to reconcile with abstract explanations. A proponent of the ontic view, then, cannot avoid begging the question.[17)]

Let me move on to the problem with generalization. Here, the problem that the ontic view encounters is that it conceives of an explanation as a particular and localized relation between concrete entities.[18)] Thus, according to the ontic view, an explanation cannot be generally applied to a range of phenomena. Can the ontic view revise its conception of explanation to accommodate generality within the ontic framework?

The first option could be to pluralize relata, especially, explanandum. In this approach, for a general explanation, while some parts of explanans explain one phenomenon, other parts explain another phenomenon. A proponent of the ontic view may add that a general explanation is a set of ontic explanations about multiple phenomena, of which members can be somehow said in a breath; this option is, however, a nonstarter, since such a set would not constitute a genuinely general explanation. A genuinely general explanation should be capable of being applied to phenomena that might not be conceived at the time the explanation is given. But, in this approach, a phenomenon that was not contemplated when a general explanation was formed, would not be part of the set of explanations. This is not what we commonly mean by calling an explanation as a 'general'

17) Thanks to an anonymous referee for suggesting a significant revision and elaboration of the problem.

18) Literatures on this debate often deal with this feature of the ontic view as if it excludes non-causal explanations. But, as mentioned in footnote 15, I think the ontic view can be extended to embrace non-causal explanatory relations, such as grounding. For instance, when the molecular structure of a given glass grounds its fragility, a proponent of the ontic view may identify this particular and localized grounding relation with the explanation of the fragility.

one. As Sheredos (2016) notes, the generality of an explanation does not come from the extension of its applications, but from how we categorize relevant items within our own terms. And these categorical terms occur in our languages as predicates, each of which cannot be replaced with the list of names to which it is applied.[19] Thus, a set of explanations is never identical with a general explanation, even when the extension of the former overlaps with that of the latter.

The second option would be to commit to universals which can be meant by our categorical terms, predicates; but this approach resurects the problem of question-begging mentioned when I dealt with the norm of abstraction. Note that universals are introduced as relata of general explanations. A proponent of the ontic view, in light of the view, may replace statements like "A system's getting more sunlight *generally explains* an increase of temperature in it" with something like: "(A system's having) the universal getting more sunlight explains (the system's having) the universal increasing temperature." Again, she must be ontologically committed to such universals in order to assert that the general explanation is also ontic. Simultaneously, her only rationale to be committed to the universals is to appeal to the bestness of the general explanation. But the bestness of the general explanation does not promise the existence of relevant universals, since it is possible that the best explanation be simply deemed as a complex of representations.

The third and final option, which has been neglected in the literature, is to embrace property realism without invoking universals. For instance, one can define a property as a *natural* class of things (or, of "*possibilia*"; see Lewis 1983 for details). This move is not the same thing with the first option, since it allows to quantify over classes, as properties with

19) Sheredos (2016), pp. 935-6.

generality[20]; unfortunately, I cannot help but repeating the same sort of problem, with a slight revision. Suppose that we have a general explanation designated by the sentence 'C_A explains C_B.' A proponent of this version of the ontic view may be tempted to say that the terms 'C_A' and 'C_B' refer to some natural classes. Again, in order to be ontologically committed to such (natural) classes, she should appeal to the bestness of the explanation. However, again, the bestness of the explanation does not ensure the existence of such classes. (Similar remarks can be said for any approach from property realism without invoking universals.)

Here, one may be intrigued whether the epistemic view can circumvent this ontological commitment to classes. We have seen that the ontic view regards an explanation as a *complex* of factive entities, while the epistemic view deems it as a *complex* of representations. And examining the ontic view, I have used the term 'complex' as if it is interchangable with the term 'relation.' One may have an impression that a proponent of the epistemic view should also be ontologically committed to relations and classes, insofar as we understand the notions of relation and class in set-theoretical terms.

Fortunately, proponents of the epistemic view are allowed to take the term in ontologically neutral manners. The idea is inspired by the debate on the logical form of grounding, which is closely related to the notion of metaphysical explanation. In the ongoing debate, the question is this: "which is the best vocabulary to regiment the notion of grounding?" There are at least three viewpoints. The first one takes the verb 'ground' as the best one, so that understands the notion of grounding as a relation signified by the verb predicate. Among others, only this first viewpoint permits a factive entity to ground another. The second viewpoint suggests that the sentential connective 'because' is the most suitable. The third viewpoint

20) Let me be neutral to the question of whether or not this move calls for a higher-order quantifier logic to quantify over classes.

favors the sentence-forming operator 'in virtue of.' In essence, the last two viewpoints only allow linguistic items to ground other linguistic items, remaining neutral regarding ontological commitments to relations (for a general introduction to this topic, see Trogdon 2013: sect. 3, and also Fine 2012 for an endorsement of the connective view). We can apply this idea to the epistemic view of explanation (regardless of which one is the correct vocabulary to regiment the notion of grounding). That is, if a proponent of the epistemic view is averse to being committed to relations, she can interpret the term 'complex' as denoting either a compound sentence formed by the sentential connective 'because', or a sentence with the operator 'in virtue of.' These vocabularies align with explanation-related talks as well as grounding-related talks.

Consequently, neither abstract explanations nor general explanations are subsumed under the ontic conception of explanation. This point is crucial, even for those who uphold only the normative priority claim of the ontic view. Since the ontic view cannot accommodate abstract and general explanations, there is nothing to which abstract and general explanations in an epistemic sense *should* correspond. (Note that this is not claiming that such explanations *do not* or *cannot* correspond to anything). So, if the ontic view were correct, abstract and general explanations cannot be assessed at all. This consequence is unacceptable. Hence, either there is no corresponding ontic explanation for abstract or general explanations, or it is not the case that abstract or general explanations must always be assessed in light of their correspondence to some ontic explanations. I will refer to this conclusion as 'the deregulation' from here on; the label reflects on its less restriction compared with the ontic norm, which sounds like a somewhat reasonable regulation for explanations.

One might wonder, then, how to assess abstract and general explanations, as legitimate ones. So far, an assessment of an explanation has been described in terms of accepting or denying its legitimacy, with 'legitimacy'

used as a somewhat vague and abstract term. I introduce this term not to make the dispute surrounding assessments of explanations into a merely verbal one, or a trivial corollary from the conceptual disagreement over explanations. Those who adhere to the ontic norm, which suggests that an explanatory text and the like should corresponds to the relevant ontic relation, seem to sympathize that an explanation, at first, should be true. However, proponents of the epistemic view may not necessarily agree. For them, as noted above, an explanatory model need not be strictly square with the target phenomenon. Moreover, some of them claims that even a *fictionalized* model can explain[21]. These proponents may emphasize explanations' usefulness, unification, communicative efficiency, and the like. So, it is highly probable that the disagreement on subjects for assessment stems from the fact that their senses of explanation differ. If you cannot dispel the thought that the disagreement is definitely a verbal dispute naturally derived from the conceptual disagreement, I have done my work in this section. I have argued that the ontic view of explanation is unsustainable, so we have no right to require an abstract or general explanation to correspond to an ontic relation. This ends up with that an explanation legitimate-from-an-epistemic-perspective can only have thin ontological imports. That's it.

But let's be more cautious and suppose that there is a common ground for assessment between the two camps. I'm denoting this common ground by the term 'legitimacy' here. I do not expect I can articulate what it is, or demonstrate whether there really is such a common ground at all. However, based upon the deregulation, I can say one thing: being legitimate is not equivalent to being *true*.[22]

21) Bokulich (2011)

22) If being a legitimate explanation is not equivalent to being a true one, some versions of the epistemic view which grasp items of explanations as knowledge turn out to be naively inclined to the ontic norm.

Intuitively, it might be most appealing to require a legitimate explanation to be *true*, especially for particular explanations. But when it comes to abstract or general explanations, this requirement conflicts with the deregulation, so long as we insist on the notion of truth as correspondence.[23] We should choose between the requirement and the deregulation. I argue that it is far more cheap to choose the latter. For abandoning the deregulation is simply reverting to the ontic view of explanation. I have discussed why it is uncomfortable. In contrast, the requirement is resulted by a hasty shift from the truism that an explanation should be about worldly facts or objects. It is dogmatic to think that the notion of aboutness is best understood in terms of correspondence or reference, and then, with the truism, an explanation should be true. It is true that, in many cases, this shift seems natural. A representation about *the* tides must refer to some actual events, a representation about *the* causal relation between the gravitational attraction of the moon and the tides must refers to an actual causal mechanism, and so on. But it is not the case that every representation *about* something is referring to that, since we have representations about nonexistence, fictional characters, and putative posits. More importantly, even abstract, fictionalized, or putative explanations are *about* somethings. At least, they are informative about target phenomena, in the sense that they can contribute to reduce our uncertainty regarding them.

The point of the previous three paragraphs is this: Renouncing the requirement does not mean that an abstract or general explanation cannot be true. The deregulation, in principle, does not preclude the possibility that such an explanation corresponds to something in reality. But once

23) I do not want to discuss whether or not we can abandon this orthodox view of truth as correspondence. But anyway, if an alternative view of truth turned out to be a possible option, it would never be a bad news to the epistemic camp. After all, what the deregulation contradicts is the notion of truth *as correspondence*,

we renounce the requirement, whether it is *metaphysically* possible that a legitimate explanation is assessed as true depends on what we are ontologically committed to, not *vice versa*. Thus, we cannot infer the existence of entities from the legitimacy of an explanation referring to them.

To be fair, all the things allude that, for cases of singular explanations of particular phenomena, the normative priority claim of the ontic view may be correct. Admittedly, the requirement that an explanation to be legitimate, successful, or true should reflect an relation between ontic items *insofar as such a thing exists*, is appealing. And singular explanations which are legitimate, successful, or true, may firmly have corresponding ontic relations. This is why several philosophers explicitly endorse or implicitly presume the normative priority of the ontic sense. For instance, Salmon (1989), Glennan (2011), Craver 2014) and so on, as had mentioned so far, explicitly endorse it; while Strevens (2018), Barrantes (2020), Kuorikoski (2021) and so on, in some contexts of other debates, presume this view whether methodologically or unconsciously. And even some proponents of epistemic view acknowledge such a normative priority of the ontic sense in particular explanations. They seek to "reconcile" the two views by partitioning realm to the part of which each sense possesses the normative priority over the opponent sense (Sheredos 2016, manuscript); though some others see as "scientifically interesting explanations are almost always explanations of classes of events."[24)]

But this putative limitation of the epistemic view may not disturb my ultimate goal in this paper. Because I will establish that *every* mathematical explanation is a general explanation, even when it aims to explain a particular phenomenon. If this argument holds, whether or not the ontic sense has its own realm becomes an irrelevant issue to my point.

24) Wright and van Eck (2018), p. 1020.

3. Mathematical explanations and the epistemic conception of explanation

According to the Quinean criterion of ontological commitment, we ought to be ontologically committed to all and only entities that are indispensable to our best theory. Here, the notion of indispensability is not simply about an entity's inability to be removed. An entity is indispensible to a theory if and only if we cannot obtain any better theory when we remove the entity. This formulation, however, is somewhat ambiguous, since it is silent to in what respect a theory could be better than others. Colyvan (2019) refines this formulation by clarifying that an entity should be *explanatorily* indispensable for us to be ontologically committed to it.

Now, imagine a situation in which a mother tries to distribute 23 strawberries evnely among her 3 children without cutting or grinding them. Definitely, her trials consistently fail. How to explain this impossibility? One would say that "it is *because* 3 is not a divisor of 23." In this toy example suggested by Braine (1972), it seems that the fact *3 is not a divisor of 23* is part of the explanans. Furthermore, for the sake of convenience, suppose that this is the best explanation of the impossibility. Then, the mathematical fact and the entities are indispensable. This commitment leads us to the existence of mathematical fact and entities.

For mathematical realists who follow this line of argument, the remaining tasks to hold mathematical realism are finding out legitimate mathematical explanations and showing the bestness of such explanations, which were merely supposed in the above example. Since various mature fields of natural and social sciences largely depend upon mathematics, the latter task may not be overly challenging. It is common in several fields of sciences, that our inquiries are about precise quantities, and it is not the case that a Field-like translation does always work well. (See Dorato and Felline 2009: sect. 3. They do not endorse mathematical realism, but the

bestness of some mathematical explanations.) Hence, it is not surprising that the mathematical realists are devoted to establishing legitimate mathematical explanations.

In contrast, some mathematical nominalists attempt to undercut this argument by showing that alleged mathematical explanations, in fact, do not necessarily require mathematical facts or objects to be part of explanans. They contend that, in so-called mathematical explanations which realists delve into establishing, mathematics merely serves as a tool for expressions, and what really carry out explanatory roles are the things that are expressed by such an expressive tool.[25] For instance, against realists claiming that "the explanation of [the Kirkwood gaps] is provided by the mathematics of eigenvalues"[26], Bueno says that

> the eigenvalues of the system are not what explain that behaviour. Rather such values emerge from the particular physical interactions among the objects that characterize the system, as long as the mathematics used to describe the system is interpreted in a suitable way. (Bueno 2012, p. 973)

The debate is based upon the ontic sense of explanation. For the mathematical realists following the line of argument described above, the explanatory significance of mathematics should be able to ensure the existence of mathematical facts or objects which themselves *explain* target phenomena. The same assumption can be found among the nominalists as well. It is obvious that the refutation of explanatoriness of mathematical explanations presupposes, and even endorse that explanatory relations hold between what are expressed by representations. The nominalists strictly distinguish the *expressive* role that mathematics can perform in

25) Bueno (2012); Strevens (2018); Barrantes (2020), p. 601.
26) Colyvan (2010), p. 302.

explanations of physical phenomena from the explanatory role that should be performed by "physical interactions among objects."

This whole debate would turn out to be misguided from the beginning if the ontic conception of explanation were ultimately unsustainable. According to *No Foundation Objection*, statements of the form 'A explains B', where 'A' and 'B' denote distinct factive entities, can have only idiomatic senses. So, the statement 'that 3 is not a divisor of 23 explains the mother's necessary failure of even distribution of 23 strawberries to her 3 children' means, in appropriate contexts, that mathematicians explain the phenomenon, appealing to the formalism of number theory from which a representation of that *3 is not a divisor of 23* is derived. Although this example allows the representation to be of something, this is not necessary. For it is possible that a representation flashes into us *ex nihilo*. We are even able to possess some representations which stand for necessary falsehoods. And some of them are explanatorily indispensable to our best theory, since reductio ad absurdum is an important method of most, if not all, fields of empirical science and mathematics. Consequently, if the epistemic sense is the sole sense of explanation, there is no room for mathematical facts or objects to play explanatory roles in their own rights. And then, the legitimacy of mathematical explanations has nothing to do with mathematical realism.

As mentioned in the previous section, however, *No Foundation Objection* is not determinate. Moreover, neither is *Narrow Conception Objection* sufficient to demonstrate that mathematical explanations do not imply ontological commitments of mathematical facts or objects. For mathematical realists who appeal to explanatory indispensability of mathematics could found their sanctuary where some mathematical explanations are legitimate but neither abstract nor general. That is, there might be singular explanation of particular phenomena. In these cases, the alleged ontic explanations are supposed to be normatively prior

than explanatory representations of any kind, so that the existence of mathematical facts or objects is required for the explanations to obey the ontic norm.

What is needed to compete with this possibility is to guarantee that *every* mathematical explanation satisfies the norm of abstraction, generalization, or both. At a glance, it seems trivial. Even if we ought to be committed to mathematical facts or objects, they are supposed to be abstract entities (for mathematical platonists), or universals (for Aristotelian realists); if not, there would be no barrier for mathematical expressions to be abstract or general. However, precisely how does this make sure that mathematical explanations are abstract or general? After all, some instances of mathematical explanations in the market seems like very specific ones, with respect to their explanatory targets. For instance, consider the case of Königsberg bridge problem and Euler's solution, offered by Pincock (2007). In 18th century, there were seven bridges connecting four landmasses in Königsberg. Denoting each landmass as L_A, L_B, L_C, and L_D, two of them linked L_A with L_C, other two of them linked L_A with L_B, and each of the rests linked L_D with L_A, L_B, and L_C, respectively. The inhabitants of the city wondered whether there was any walkway over all the seven bridges without retracing. The great mathematician Leonard Euler gave the solution with the mathematical statement which, today, amounts to that there is no Eulerian path for the graph where the road system of the city is considered as a multigraph, the bridges as edges, and the landmasses as nodes, in the graph theoretic terms. This solution, with his proof for that statement, explained why it is impossible to walk over all the bridges without doubling back. Here, the description of the explanandum is mentioning the proper name of the city in the real world, with depicting its road system specifically (though my description abstracted away some details). Isn't it a case of particular mathematical

explanation, which mathematical realists hope to assess by comparing with the ontic relation consisting of the mathematical *fact* that Euler proved and the particular empirical fact about the road system?

To reply to this question, let me concentrate on generality, and let's think about under what condition we call an explanation as 'particular' or 'general', in causal explanations. If events of type C regularly cause events of type E, so that if [the representation of] event type C explains [the presence of] event type E, this explanation is obviously general. Conversely, if event token c causes event token e, so that [the representation of] c explains [the presence of] e, this explanation is obviously particular. Calling the former as 'a type-type explanation' and the latter as 'a token-token explanation', what would be said about, namely, type-token explanations and token-type explanations? Here, I'm taking these cases metaphorically, since it is unclear what the cases of an event type causing an event token, and of an event token causing an event type are. So, spot me to stipulate a type-token casual explanation to be an explanation of an event token by accommodating it under some causal governing-laws. And let a token-type causal explanation be an explanation in which an event token causes a bunch of events of the same type.

I argue that type-token explanations are general in spite of its particular explanandum, and that token-type explanations are particular in spite of its psuedo-general explanandum (strictly speaking, an array of explananda). These might be shown by offering concrete examples of each cases. Presumably, it amounts to a type-token explanation that, for a given specific earthquake, an geological model or fact of plate tectonics explains the earthquake, perhaps, invoking a stress accumulation due to friction of plate boundaries which can make the rocks elastic, and then slip. This line of explanation is likely general, since we know that it can be applied to other, possibly to all, occurrences of earthquake token, though it is not being applied so. In other words, this explanation has explanatory depth.

On the other hand, let's consider some effects of the earthquake. For instance, there must be changes of seismographs recoded in several areas. Assuming that those seismographs accord to the same operations, the earthquake itself, or a model of it, will explain the graphs of the same type. But, at any rate, this explanation is far from being general, since it lacks explanatory depth that makes it appliable to other graph tokens resulted by other earthquake tokens. Now, combined with the more obvious type-type case and token-token case, these observations inform us that the vehicle of particularity or generality of an explanation is its explanans, rather than explanandum. For a causal explanation seems general wherever its explanans is an event type, and seems particular wherever its explanans is an event token.

Analogously, we might be able to assume, in favor of mathematical realists, that a mathematical explanation is general if and only if its explanans is a mathematical fact (or object) type, and particular if and only if its explanans is a mathematical fact (or object) token. However, is it possible for a mathematical fact or object to be a token? Traditionally, tokens are said to be spatiotemporal, concrete and particular things, while it is said to be that types are abstract and "have no unique spatio-temporal location and therefore cannot consist of particulars, of tokens[27]." If correct, to my knowledge, there is no lively option to understand mathematical facts or objects as tokens. Consequently, for any alleged mathematical-ontic explanation, of which focal explanans is a mathematical fact or object, it must be a general explanation.

However, we have seen that, for a general explanation, it is either that there is no corresponding ontic explanation at all, or that it is not the case that abstract or general explanations should always be assessed in light of their correspondence to some ontic explanations. Hence, the legitimacy of

27) Wetzel (2018)

a mathematical explanation (in an epistemic sense) need not be assessed by checking whether it is apt to capture an ontic relation between a mathematical fact and a concrete fact. This implies that, we cannot infer from the legitimacy of mathematical explanations the existence of relevant mathematical facts.[28)]

Admittedly, the facts that the notions of type-token explanation and token-type explanation are just metaphoric, and that my stipulations of them are somehow arbitrary, would undermine the validity of my argument. Even I myself do not think that these metaphors have any significance to the disputes surrounding the account of explanation. That is, they will not provide us with any novel classification of explanations in terms of token/type distinction. Nevertheless, I believe those metaphors can be taken as a thought experiment in which we control the logical structure of the conception of explanation as if we could. And I intended to reveal, via this thought experiment, what would most likely carry particularity/generality, or, in other words, explanatory depth, in an explanation.

4. What I did argue, and what I did not.

I construe the dispute between mathematical realists who appeal to explanatory indispensability of mathematics, via seeking for genuine mathematical explanations, and mathematical nominalists who explicitly deny legitimacy of mathematical explanations competing with their enemies, as based upon the presumption of the ontic sense of explanation.

28) Be careful not to confuse this conclusion as excluding the possibility of the existence of mathematical facts or objects at all. What it does exclude is the possibility of an ontic explanation having mathematical facts or objects as its relata, and then, the validity of inference from a successful mathematical explanation the existence of mathematical facts or objects.

I began with introducing and refining several challenges that proponents of the ontic view face. For them, at worst, it might be just illusionary impression that there is any ontic sense of explanation. At best, they can salvage the ontic conception by preparing its own realm to assess explanations in epistemic senses. That is, it might be the case that the ontic conception of explanation exists independently from other epistemic senses, and, in its own realm, it is normatively prior in assessing explanatory representations, by checking whether such representations are apt to capture the relevant ontic explanatory relations. However, even if correct, the normative priority of the ontic sense is restricted to the case of particular explanations held between concrete entities. I argued that generality/particularity, or, in other words, explanatory depth of an explanation is carried by its explanans. More precisely, an explanation is general if and only if its explanans is a type, and particular if and only if its explanans is a token. If then, a mathematical explanation cannot be a particular explanation, since its focal explanans, a mathematical fact, cannot be understood as a token. Consequently, the legitimacy of a mathematical explanation does not hinge onto whether it captures the relevant relation between mathematical and concrete facts. And then, we cannot infer the existence of mathematical facts from the legitimacy of mathematical explanations.

I have a caveat about what I did not argue. And I hope this remark to be (meta-)ontological significance of this paper:

I do not think that I have argued that the legitimacy of an explanation is irrelevant to ontological commitments. In short, what I have asserted is that an explanation's legitimacy can justify beliefs in the existence of relevant entities only when it (metaphysically) can be assessed as true (as correspondence). And whether a mathematical explanation amounts to such a case depends on the assessor's ontological commitment, not *vice versa*.

Let's compare this with the inference to the best explanations of scientific realism. Even those who do not concede the validity of the inference, scientific anti-realists, may accept that it is metaphysically possible that our current scientific theories would be assessed as true were we (physically) able to observe atoms, black holes, and so on. Thus, regarding such material-but-unobservable things, an explanation's legitimacy, successfulness, bestness, or whatever, may serve as a good reason to believe in the existence of them. But this circumstance is not something usual in metaphysical disputes. A realist and an anti-realist of a metaphysical entity cannot concur in possibility of assessments-as-true about explanations in which such an entity is invoked. The entity either necessarily exists, or necessarily not.

I think this remarks the difference between what a legitimate explanation can and cannot have as its ontological imports. If it is metaphysically possible that an explanation's legitimacy amounts to its truth, the inference from the legitimacy to the existence of relevant entities is more or less reasonable. If not, an explanation's legitimacy is autonomous from its truth, ontological commitments, and so on.

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수학적 설명에 대한 인식적 관점과 그 존재론적 함의

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일부 수학적 실재론자들은 경험적 현상에 대한 수학적 설명의 적법성을 확립하고자 한다. 그들에 따르면, 만일 경험적 현상에 대한 적법한 수학적 설명이 존재하고, 나아가 그것이 관련된 현상에 대한 최선의 설명으로 인정된다면, 우리는 그러한 수학적 설명에서 사용된 수학적 사실이나 대상에 대한 존재 믿음을 가져야 한다. 이에 대한 반동으로, 일부 수학적 유명론자들은 그와 같은 수학적 설명의 적법성을 거부하길 시도한다. 본고는 수학적 설명의 적법성을 둘러싼 이들 사이의 논쟁이 설명에 관한 그릇되거나, 혹은 지나치게 협소한 개념인 존재적 개념에 바탕을 두고 있다고 주장한다. 만일 우리가 이를 거부하고 설명에 관한 인식적 개념을 수용한다면, 수학적 설명의 적법성을 받아들이는 일은 더 이상 수학적 사실이나 대상에 대한 존재 믿음을 강제하지 않을 것이다.

주요어: 과학적 설명, 설명에 관한 인식적 개념, 수학적 설명, 수학적 실재론, 불가결성 논제